1. Two spins

The normalized spin state of two spin $\frac{1}{2}$ particles is given by

$$a \uparrow \uparrow_1 \uparrow \uparrow_2 + b \uparrow \downarrow_1 \downarrow \uparrow_2 + c \downarrow \uparrow_1 \uparrow \downarrow_2$$

A measurement gives that the spin of particle 1 is up. Afterwards, what is the probability that particle 2 has spin down?

2. Fermions in a square

Consider $2N$ non-interacting fermions moving in two dimensions, inside a square box with sides of length $L$. The fermions have mass $m$ and move non-relativistically. $N$ is large. Compute the Fermi energy.

3. Shaken oscillator

Consider a single harmonic oscillator with frequency $\omega$ and a small time-dependent perturbation

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \epsilon(t) x$$

with

$$\epsilon(t) = \begin{cases} 0 & t < 0 \\ \epsilon & 0 < t < T \\ 0 & T < t \end{cases}$$

The particle starts out in the ground state.

(a) Compute the expectation value of $x$ as a function of time $t < T$.

(b) Using first order perturbation theory, find the transition probability for the particle to end up in the first excited state. Explain what happens for $T = \frac{2\pi}{\omega}$. What is the energy of the final state?