# Physics 103

Department of Physics
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## Precept Notes

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## Rigid Body Motion and Statics

## Part 2

# Announcements

- Finish Chapters 13 and 12 this week.
- There is an additional reading from Young & Freedman on gyroscopes in e-reserves (blackboard).
- Problem 4b is ok after all.
- The exams are graded: average 65%. You must give them to me before leaving if you want a regrade.

## Key to Exam Success

- To do well on the exam, you have to think of convincing the grader your result is correct. Excessively brief explanations, unreadable solutions, or haphazard organization lose points.
- Don't waste time memorizing complicated formulas in the book. You won't get credit for using them. Derivations in terms of fundamental concepts show that you understand them, and get more points.

## Rolling Constraint

When a wheel is rolling, it goes a distance $s = 2\pi R = vt$ every time t it rolls once.

In the same time, it turns through an angle

$2\pi = \omega t$. Dividing gives $R = v/\omega$, or $v = R\omega$.

The axle goes at speed v.

A point on top goes at speed $v + Rw = 2v$.

A point on the ground goes at speed

$v - R\omega = 0$! That means the point on the ground is instantaneously at rest.

# Question about Problem 4

This is a technical point but the reason for the confusion about problem 4 is that some people noticed that when a wheel rolls inside a hoop, the center doesn't move as far as the edge. This invalidates the derivation of $v = R\omega$, but not the result. It is ok to just remember that this still works for a curved path.

# Rolling inside a curved Path

You can skip this slide: it is here for my own reference.

When the wheel rolls once so the same point comes back in contact with the hoop, it has traveled a distance s = 2pr along the hoop and through an angle $\theta = s/R = 2\pi r/R$.

The cm has moved a distance

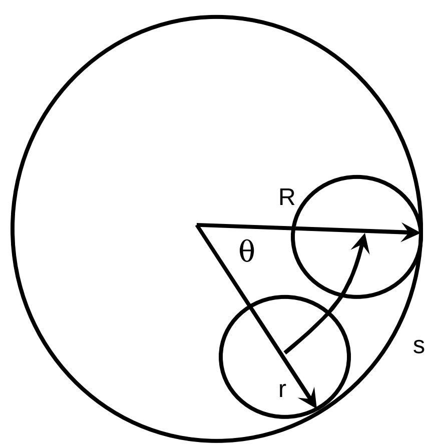
$$(R-r)\theta = 2\pi r(R-r)/R = vt.$$

The wheel has rotated through an angle*

$$2\pi - \theta = 2\pi(1 - r/R) = \omega t.$$

Dividing again gives

$$v/\omega = r$$




*The direction from the center to the edge has rotated counterclockwise by angle $\theta$.

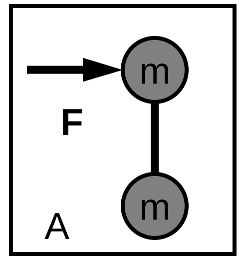
# Dumbbell

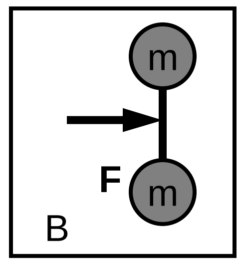
A force F is applied for time t to a dumbbell in one of two ways shown.

Which gives the greater speed to the center of mass?

(a) A (b) B

(c) the same




# Rigid Body Motion

Force causes the CM of an object to accelerate:

$\mathbf{F} = m\mathbf{a}$.

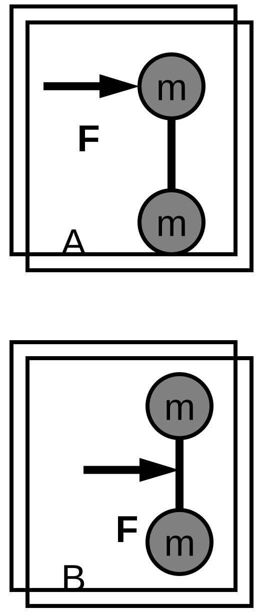
Torque causes rotation of the object about the CM:

$\tau = I\,\alpha$.

For now we will consider only "fixed axis rotation", where the direction of the axis doesn't change. Later, we will generalize, partially. A complete treatment of general rotational motion is beyond the scope of this course.

# Dumbbell

- The change in the linear momentum of the center of mass is $\Delta p = Ft$ in either case.
- It doesn't matter where the force is applied. Only the magnitude and direction affect the change in linear momentum.



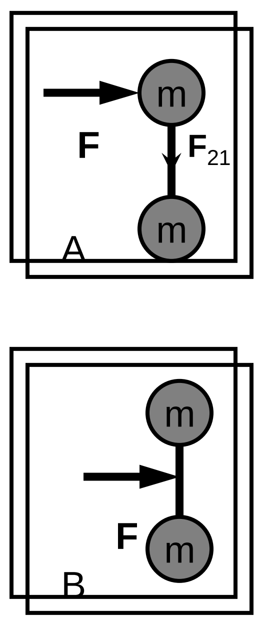

# Dumbbell

In case B, pushing at the CM gives no rotation, $F = 2ma$, $mv = Ft/2$.

In case A, pushing on the top mass gives it initial acceleration $a = F/m$.

As the dumbbell begins to rotate, the bottom mass will begin to pull down on the top one with force $\mathbf{F}_{21}$ to pull it around in a circle.

But for a short time t, the motion will be purely horizontal.



## Dumbbell

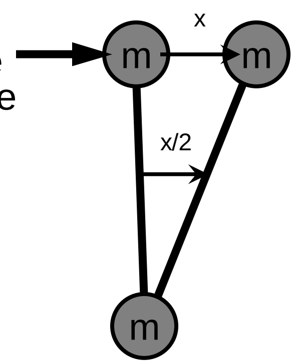
For a small time,

$mv = Ft$, at the top.

The bottom mass doesn't move at all, instantaneously: it is the initial rotation axis. (The rod could only pull it up or down, not sideways, at first.)

At the center, $v_{cm} = v/2$ since it goes half as far.

$v_{cm} = Ft/2m$ as in case B.




## Dumbbell

Which force does more work?

(a) A (b) B

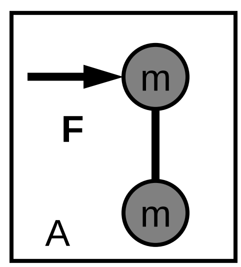
(c) the same

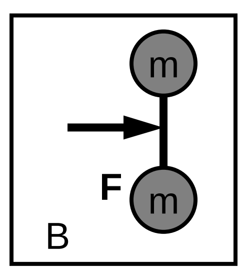
A) $W = ½ (2m)v^2 + ½ I\omega^2$

$= mv^2 + ½ (2mR^2)(v/R)^2$

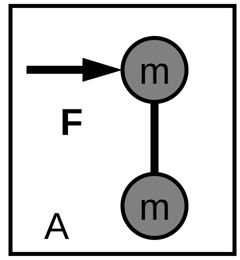
$= 2mv^2$.

B) $W = mv^2$.



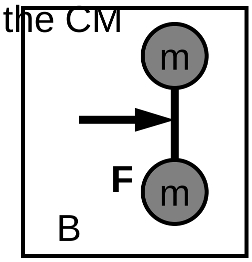



## Dumbbell




That means twice as much work must be done in case A to give the CM the same speed as in case B.

That is right, because the top mass must be pushed twice as far to get the CM to move the same amount initially.

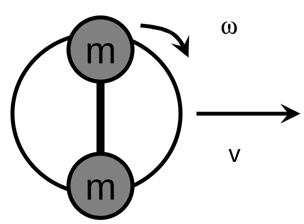



## Dumbbell

After the initial push, the two masses will continue moving to the right at speed $v = Ft/m$, and rotating about the center with $\omega = \tau t/I = v/R$.

The initial speed of the top mass is a superposition, $v_{top} = v + R\omega = 2v$ as we found before.
The initial speed of the bottom mass is $v_{bottom} = v - R\omega = 0$.

This relation between $R\omega$ and $v$ is a consequence of the moment of inertia – different objects will behave differently.

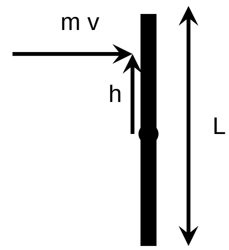


## Bullet and Stick

Momentum is conserved:

$mv = MV + mv/2$

$V = mv/2M$.




Angular momentum is conserved:

$mvh = I\omega + mvh/2$

$\frac{1}{2}\, mvh = ML^2\, \omega / 12$

$\omega = 6\, mvh/ML^2 = 12\, Vh/L^2$.

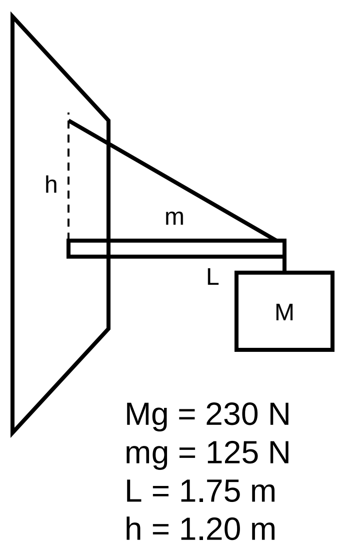
## Sign Problem

Because the exams were returned, we didn't get to this example, but it might be useful…

A sign weighing 230 N is supported by a 90 N beam of length 1.75 m.

The guy wire is attached to a point 1.20 m above the beam.

Find the tension in the guy wire and the force of the wall on the beam.




$Mg = 230$ N
$mg = 125$ N
$L = 1.75$ m
$h = 1.20$ m

## Sign Problem

Mg = 230 N
mg = 90 N
L = 1.75 m
h = 1.20 m

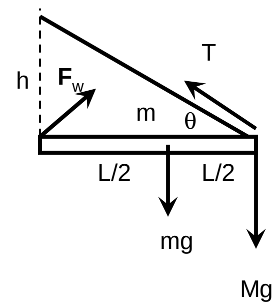
Torque about left end of beam:

$\tau$ = LT sin q – MgL – mgL/2 = 0.

T = (M + m/2)g / sin q

tan $\theta$ = h/L = 0.6857, $\theta$ = 34.4°, sin $\theta$ = 0.565.

T = 275 N / 0.565 = 487 N.




## Sign Problem

Mg = 230 N
mg = 90 N
L = 1.75 m
h = 1.20 m
T = 487 N

$F_{wx}$ = T cos $\theta$ = 402 N.

Torque about sign end:

$LF_{wy}$ = Lmg/2.

$F_{wy}$ = mg/2 = 45 N.

Fw = 405 N, $\phi$ = 6.39°.