Physics 103

Department of Physics
Princeton University

Precept Notes

S. Yost
November 5, 2007

Conservative Forces and Gravity

Part 2

Exam 2 Review

Announcements

- Problem set 6b is due tonight.
- Exam 2 is tomorrow: featuring Energy and Momentum
- Start a new topic: Rigid Bodies, Knight Chapter 13.
- Learning guides: LG3, LG4, LG7
- 2005 and 2006 exams.
- 2006 exam 2 is particularly good for review.
- 2006 final, problems 2(a) and 5 (a-c).
- 2005 quiz 3, 4.

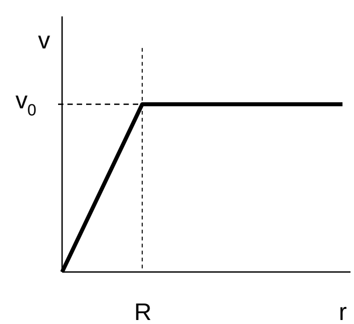
Main Points on Exam

- Momentum and Impulse
- Elastic and Inelastic Collisions
- 2D Collisions
- Center of Mass Motion, Center of Mass Frame
- Work, Energy and Power
- Gravitational Force and Energy

Galactic Rotation Curve

An idealized but realistic galactic rotation curve is shown.

Assuming a spherical galaxy, what mass density $\rho(r)$ would give the rotation curve for $r < R$?



Galactic Rotation Curve

A distance r from the center,

$$a = GM(r)/r^2 = v^2/r$$

$$M(r) = \int_{r'<r} 4\pi r'^2 \rho(r')dr'.$$

Centripetal force. v is proportional to r, so $v = v_0 r/R$, $r < R$, and

$$(v_0/R)^2 r^3 = GM(r)$$

This implies $\rho(r') = \rho =$ constant, with

$$GM(R) = 4\pi G R^3 \rho/3 = v_0^2 R.$$

Thus $\rho = 3v_0^2/4\pi G R^2 = \rho_0$.

Galactic Rotation Curve

What spherical mass distribution would produce the curve for $r > R$?

$v = v_0 =$ constant implies

$$GM(r) = v_0^2\, r \quad \text{for } r > R.$$

$GM(r) = \int_{r'<r} 4\pi G r'^2 \rho(r')dr' = v_0^2 r$

Need $\rho(r') = c/r'^2$ so that

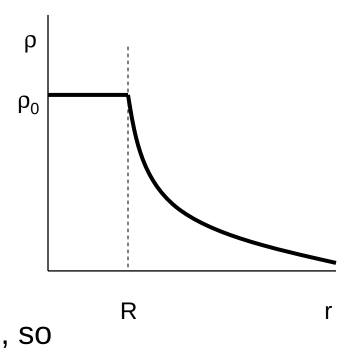
$$GM(r) = GM(R) + 4\pi Gc(r - R)$$

$$GM(r) = v_0^2 r$$

$$= v_0^2 R + 4\pi Gc(r - R).$$

$c = v_0^2/4\pi G$ gives the correct relation, so

$$\rho(r) = v_0^2/4\pi G r^2 \quad \text{for } r > R.$$



Galactic Rotation Curve

Compare this to the luminous matter density

$L = L_0 \, e^{-r/r0}$ and the flat part of the rotation curve extends beyond the visible stars in a galaxy.

What does this suggest about the visible mass of a galaxy. [Dark matter.]

Energy Released in Collision

Two cars collide inelastically so that the total energy $K = K_1 + K_2$ is fixed. What division of energy between the two cars is likely to cause the most damage in the collision?

Energy Released in Collision

Which collision releases more energy? To quantify it, think of the cars as having a bumper that gets compressed. Which collision does more work on the bumpers as the cars come together?

Maximize W

$K = K_f + W$. $W = K - K_f$

Energy Released in Collision

$K_i = K = K_1 + K_2$

$K_1 = p_1^2/2m_1$ $K_2 = p_2^2/2m_2$

$K_f = ½ P^2/(m_1 + m_2)$ with $P = p_1 + p_2$.

Remember that $W = K - K_f$.

K is fixed. But we can make $K_f = 0$. This gives $W = K$, the maximum possible work.

Kf = 0 if P = 0. Then $p_1^2 = p_2^2$ which implies

$m_1K_1 = m_2K_2$. So $K_1/K_2 = m_2/m_1$.

This gives $K_1 = m_2K/(m_1+m_2)$, $K_2 = m_1K/(m_1+m_2)$.

Energy Released in Collision

In general: $K = K_{cm} + K'$ where K' is the energy in the CM frame. Since K_{cm} is constant throughout a collision, the maximum energy available to do work in the collision is found when $K_{cm} = 0$. This would be true for any type of collision.

Elastic Collision

A car of mass m_1 and energy K strikes a stationary mass m_2 and has an elastic collision. Assume the cars have bouncy bumpers and different masses.

Does the maximum compression of the bumpers occur if the heavier car is moving, or the lighter one? Does it matter?

Elastic Collision

This time, we can't set set $K_{cm} = 0$ before the collision. It is fixed by the conditions

$K_{cm} = P^2/2(m_1+m_2)$ with $P^2 = 2m_1K$.

Then $K_{cm} = m_1K/(m_1+m_2)$.

$K' = K – K_{cm} = m_2K/(m_1+m_2)$ before the collision.

$K' = 0$ at maximum compression (turning point).

$W = m_2K/(m_1+m_2) = K/(1 + m_1/m_2)$. This is greatest if $m_1/m_2 < 1$, meaning that the greatest compression occurs when the lighter object is the one moving.

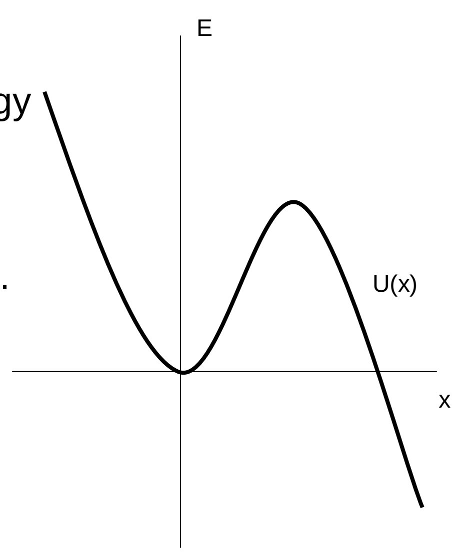
1D Potential Energy

Consider a conservative force with potential energy $U(x) = bx^2 – cx^3$.

What is F(x)?

$F(x) = – dU/dx = – 2bx + 3cx^2$.

Where are the stable and unstable equilibrium points?

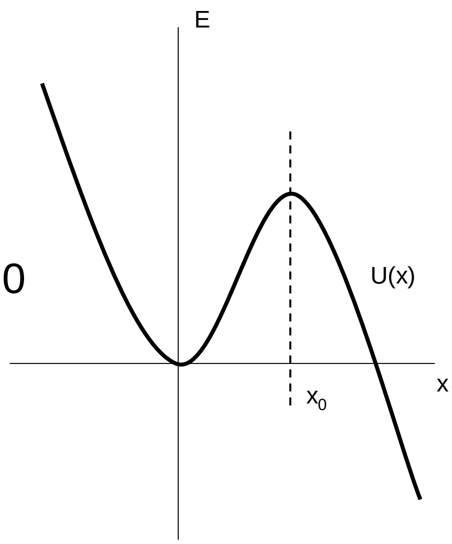


1D Potential Energy

F(x) = 0 in equilibrium.
x = 0 or 2b = 3cx.

Stable equilibrium: x = 0

Unstable equilibrium:
$x = x_0 = 2b/3c$.



1D Potential Energy

Where is the force
to the left?

Between 0 an x_0.

The force is strongest
where the slope is steepest.