Physics 103

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Precept Notes

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October 24, 2007

Conservative Forces and Gravity, Power

Part 1: Power continued (rocket), Gravitational Energy

Announcements

- This week's reading assignment is Chapters 13 and 14 of the Feynman Lectures (in E-Reserves), excluding sections 13.4 and 14.5. For the definition of the gravitational force, read the first section of Knight 12.3; you may stop just prior to the section headed Gravitational Force and Weight (on page 348).
- The exam on momentum and energy will be the Tuesday after fall break.

Feynman Lectures

- Read about the relation between forces and potential energy in 3 dimensions.
- Read about Feynman's ideas on energy conservation and related topics.
- You will see that there are not a lot of equations – these chapters are about understanding the concept of energy and energy conservation.
- The symbol T is used for kinetic energy.

Maintaining Constant Acceleration

Remember: With constant acceleration, the power increases linearly with time.

$$P(t) = Fv = Fat$$

Does this make sense?

Think of car wheel applying constant force while turning through circumference L.

$P = Fv = F(L/T) = FLf$ where $f = v/L$ = number of revolutions per second increases with v.

Work per revolution $W = FL$ is constant. $P = Wf$.

Doing Better?

Is it possible to do better?

Can you build a car that maintains constant acceleration with an engine that produces constant power?

Just changing the inner workings of the car won’t help. The work done by the engine is the source of the power driving the car. You can’t create more energy from nothing.

But you can try a completely different mechanism to avoid the limitations that come from pushing against the ground.

This example will illustrate a way in which energy can be conserved that you may not have considered, and also remind you of momentum conservation applications.

Rocket Propulsion

Consider for simplicity a rocket with a small amount of fuel, so the mass of the rocket doesn’t change much. (Good assumption for short time intervals.)

Total mass M.

Eject mass m at relative speed v_0.

Momentum conservation implies:

$$(M - m)v' + m(v' - v_0) = Mv.$$

$$mv_0 = M(v' - v) = Mat.$$

Constant acceleration $a = (m/t)v_0 \equiv \mu v_0$.

kg/s

Rocket Propulsion

Work done on rocket:

$W_r = DK_r = ½ (M – m)(v'^2 – v^2)$

$= ½ (M – m)(v' – v)(v' + v)$

$≈ mv_0 v = Ft v,$

$\text{Power} = W_r/t = \mu v_0 v = Fv$

This is what we would expect for constant acceleration: linearly increasing power.

Assumption $m << M$ is exact for t approaching 0, so power expression is exact.

Rocket Propulsion

From the point of view of someone onboard, the engine is always producing the same power: it accelerates a mass m of exhaust to speed v_0, doing work at the rate

$½ mv_0^2 / t = ½ \mu v_0^2.$

This does not increase with time.

- Where does the "extra" power come from?
- Think about the whole system.

Rocket Propulsion

Work done on exhausted fuel:

$W_e = \Delta K_e = \tfrac{1}{2} m((v'-v_0)^2 - v^2)$

$= \tfrac{1}{2} m(v'^2 - v^2 - 2v'v_0 + v_0^2)$

$= \tfrac{1}{2} m(v' - v)(v' + v) - mv'v_0 + \tfrac{1}{2} mv_0 2$

$\approx (m^2/M)\, v_0\, v - mv\, v_0 + \tfrac{1}{2} mv_0^2$

$\approx - mv\, v_0 + \tfrac{1}{2} mv_0^2 = \tfrac{1}{2} mv_0^2 - W_r$.

Power = $\tfrac{1}{2} \mu v_0^2 - P_r$

Net power = $\tfrac{1}{2} \mu v_0^2$ = power to accelerate fuel in rest frame of the rocket.

The rocket got some of its power by taking KE away from the fuel, by ejecting it backwards.

Rocket Propulsion

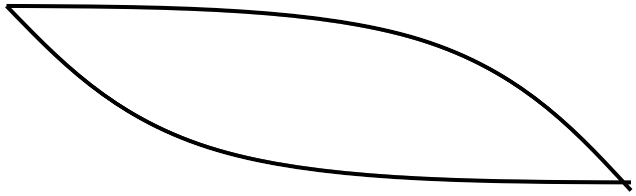
The rocket got some of its power by taking KE away from the fuel, by ejecting it backwards.

The faster the fuel is moving to begin with, the more power the rocket can get this way – it is a power source that increases with acceleration.

Lesson – energy is conserved, but you can get energy by stealing it from something else: in this case, the ejected fuel.

Two Slides

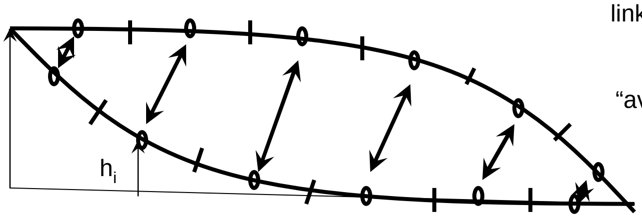
Compare the following two situations. If a box slides down each slide, which would move faster at the end? Which would take less time? The slides have the same length.



Two Slides

Gravity is conservative, so the speed at the end is determined by K = mgh either way.

The path at the bottom takes less time, because if you divide it in pieces and compare the times for segments of equal length, the times on the lower path will always be less.



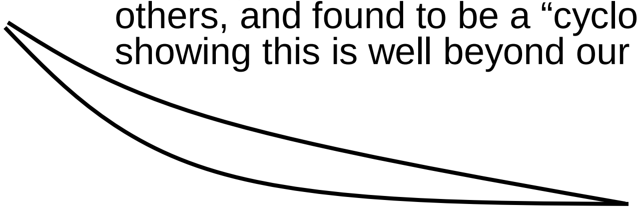
Compare times for segments linked by arrows.

Compare the "average height" of each.

Average speed per short segment: $v_i^2 = 2gh_i$

Two Slides

It is important that the slides have the same length to compare this way. For example, the case shown below could be subtle, since you have faster speeds on the lower track, but the length is longer, so it could take more time. The general problem of finding the fastest path between two points, with objects sliding only due to gravity, was solved by Newton and others, and found to be a "cycloid" shape. But showing this is well beyond our course.



Gravitational Potential Energy

The gravitational attraction is

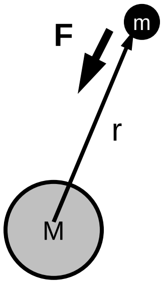
$F(r) = -G\ Mm/r^2$.

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$.

The gravitational potential energy is

$U(r) = -\ G\ Mm/r$.

The integration constant is chosen so that $U(r) = 0$ at $r = \infty$



Gravitational Potential Energy

If an object falls to Earth from rest at a very great distance, how fast does it enter the Earth's atmosphere?

"Very great distance" means infinite, or in practical terms, far enough away that it doesn't affect the result. An object infinitely far away would, in reality, never get here, so don't take this too literally.

Gravitational Potential Energy

The initial energy is zero, since $v = 0$ implies $K = 0$ and $r = \infty$ implies $U = 0$.

The final energy is then

$$0 = \frac{1}{2} mv^2 + U(R_e)$$
$$= \frac{1}{2} mv^2 - gmR_e^2/R_e$$

Gravitational Potential Energy

Since

$$0 = \frac{1}{2} mv^2 - gmR_e$$

the entry velocity is

$$v = \sqrt{2gR_e}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(6.38 \times 10^6\ \text{m})}$$
$$= 11\ \text{km/s}.$$

Gravitational Potential Energy

- Conversely,
- $v = \sqrt{2gR_e} = 11$ km/s
- is the velocity at which a projectile would have to be launched to get to $r = \infty$ with zero velocity.
- This is called the escape velocity.

Gravitational Potential Energy

For a general astronomical object, replace g by the gravitational acceleration at its surface or use, $g = GM/R^2$, to write the escape velocity in the form

$$v = \sqrt{2GM/R}$$

Gravitational Potential Energy

For the sun, $M = 2.0 \times 10^{30}$ kg and $R = 7.0 \times 10^{8}$ m.

Recalling $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

gives

$$v = \sqrt{2GM/R} = 617 \text{ km/s}$$

Gravitational Potential Energy

Laplace suggested in 1798 that a star would be black if its escape velocity were greater than the speed of light, $c = 3 \times 10^5$ km/s.

This could happen if

$$c = \sqrt{2GM/R}$$

Gravitational Potential Energy

To what radius R would the sun have to be compressed to make it a “black star”?

$$c = \sqrt{2GM/R}$$

For the actual sun

$$617 \text{ km/s} = \sqrt{2GM/R_s}$$

Gravitational Potential Energy

Divide:

$$\frac{617 \text{ km/s}}{3 \times 10^{5} \text{ km/s}} = \frac{\sqrt{2GM/R_s}}{\sqrt{2GM/R}} = \frac{\sqrt{R}}{\sqrt{R_s}}$$

Then $2.06 \times 10^{-3} = \sqrt{R/R_s}$

Gravitational Potential Energy

Since $2.06 \times 10^{-3} = \sqrt{R/R_s}$

the sun would have to be compressed to a fraction $R/R_s = 4.25 \times 10^{-6}$ of its current size to be a “black star”. This would give it a radius of

$$R = 3.0 \text{ km}.$$

Gravitational Potential Energy

In modern times, black stars are called black holes, and are predicted by Einstein's General Relativity.

In fact, the result is *exactly* the same in general relativity: the radius R = 3.0 km is the size of a black hole with the mass of the sun.

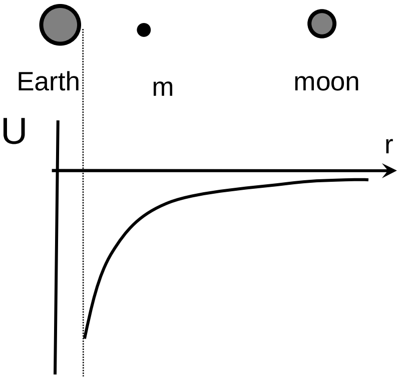
Combined Potential Energy

- When more than one gravitating body, the forces add, and so do the potential energies.
- For example, consider the earth and moon.

Combined Potential Energy

The Earth's potential energy of an object a distance r from the Earth's center is

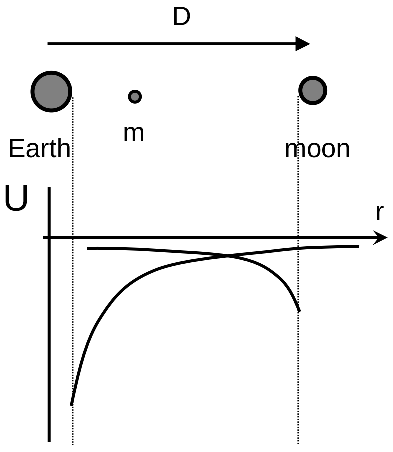
$U(r) = -GM_em/r$



Combined Potential Energy

If the moon's center is a distance D from the Earth, then in the same variable, the potential energy of the mass due to the moon's gravity is

$U(r) = -GM_mm/(D-r)$



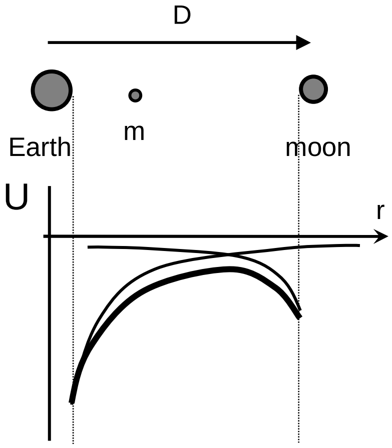
Combined Potential Energy

The potential energy anywhere between the earth and moon is the sum of these potential energies.

$U(r) = -GM_m m$

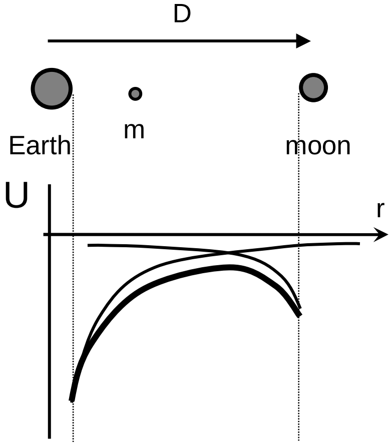
$[1/r + /(D-r)]$

$= -GM_m m\ D/r(D-r)$.



Combined Potential Energy

- The maximum potential energy is at a point where the sum of potential energies is greatest.
- Here, $dU/dr = 0$.
- Remember that $dU/dr = -F$
- The total gravitational force from the Earth and moon cancel at this point: $F_e + F_m = 0$.
- It is an unstable equilibrium point.

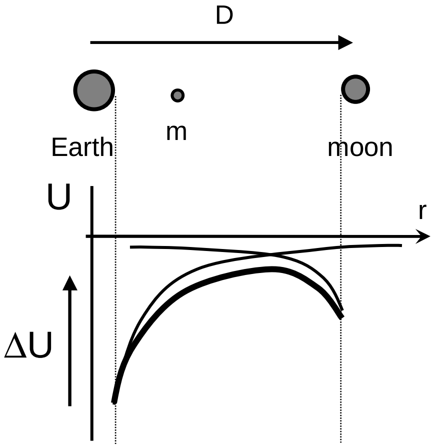


Combined Potential Energy

If a rocket is launched from the Earth and is to reach the moon, it must be given enough initial kinetic energy to get over this potential hill and reach the moon.

- $K = \Delta U$

This gets it to the top of the "hill" with zero speed.



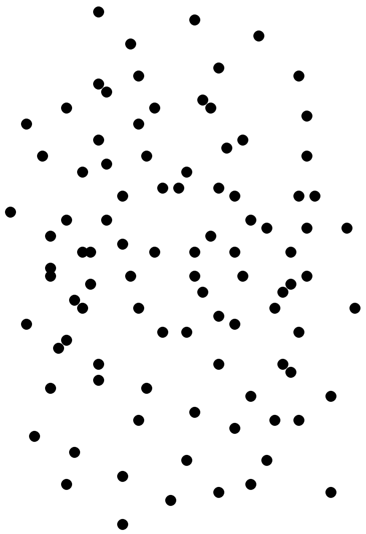
Energy in Star Formation

Energy conservation explains where the heat comes from in the early stages of star formation.

Start with a nebula containing a cold gas spread out over a large distance.

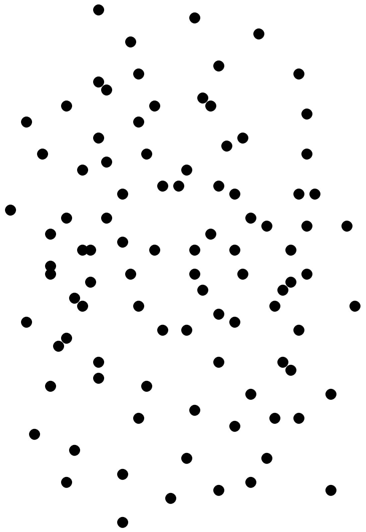
Star Formation

- The nebula originally contains gas with very low KE, and not much gravitational energy either, since the separations are relatively large.
- U + K ≈ 0 (roughly)



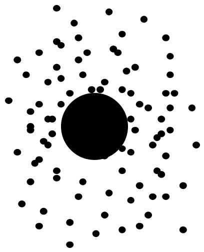
Star Formation

- The mutual gravitational attraction of the atoms pulls them together into a smaller area.
- In the process, the gravitational potential energy decreases.
- Total energy is conserved, so the atoms speed up.
- Still, U + K ≈ 0



Star Formation

- Most of the motion of the atoms will be in random directions, and this random kinetic energy is interpreted as heat in the gas.
- When the atoms are compact enough, the temperature gets high enough to start nuclear fusion, and a star is born.
- The total gravitational + kinetic energy of the star is still nearly zero! But the kinetic energy is very high, and the gravitational energy is very negative.



Star Formation

- Many kinds of energy are encountered: gravitational, kinetic, electrostatic (between the hydrogen atoms), and nuclear, as the fuse and the previously hidden binding energy inside the hydrogen nuclei is released.
- Some of this energy then radiates back into space.
- But the star doesn’t create any **new** energy – it was all there in the initial cold, diffuse gas cloud.