Physics 103

Department of Physics
Princeton University

Precept Notes

S. Yost
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Introduction to Momentum and Energy

Announcements

- The exam is being graded. The grades will be in within 1 week of the exam.
- We won't discuss the exam in class. It's time to move on – we must keep our momentum and start studying… **momentum**!
- We are discussion Chapters 9 and sec. 1, 2 and 6 from Chapter 10. Skip sec. 9.7 for now.
- Problem set 4a on Chapter 9 and parts of Chapter 10 (Kinetic Energy) is due tonight.

Force Acting Over Time

When a constant force acts on a mass over a length of time, the velocity changes.

Starting from rest,

$$F = m\,a = m\,(v/t) = mv/t.$$

Then

$$mv = F\,t.$$

We call the left-hand side the **momentum**, and the right-hand side the **impulse**.

Momentum and Impulse

- The momentum is often written as $p = mv$.
- The units of momentum are kg m/s = N s.
  - Sadly, momentum does not have a better name, in spite of being one of the most fundamental quantities in all of physics!
- Impulse is written $J = Ft$ (for constant force).
- In general, $\Delta p = \int F dt = F_{avg}\, t = J$.

More than One Dimension

In general, the velocity has a direction, so momentum is a vector: $\mathbf{p} = m\,\mathbf{v}$

So is the impulse: $\Delta\,\mathbf{p} = \int \mathbf{F}\,dt$

Newton's Law can be written in terms of momentum:

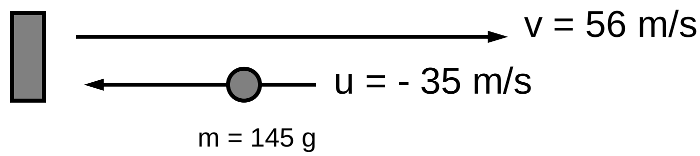
$$\mathbf{F} = d\mathbf{p}/dt.$$

This is the original form of Newton's law. He called momentum the "quantity of motion".

Example

- A 145-gram baseball is pitched at 35 m/s and hit on a line drive straight back at the pitcher at 56 m/s. If the bat was in contact with the ball for 5.0 ms, what was the average force of the bat on the ball?

Example



- It is important to keep track of the direction of the velocity: I wrote the velocity to the left as negative.

Example

The change in momentum is

$$\Delta p = m(v - u)$$
$$= 0.145 \text{ kg } (56 \text{ m/s} - (-35 \text{ m/s}))$$
$$= 13.2 \text{ kg m/s}.$$

The momentum change Δp =13.2 kg m/s is the impulse on the ball.

It occurred in time t = 0.0050 s, so the average force was

$$F = \Delta p / t = 2.64 \text{ kN}.$$

Momentum Conservation

Newton's laws were obtained in part by studying collisions between masses.

It was found that when objects collide, with no external forces, momentum is conserved.

In fact, as long as you take a big enough system that all relevant forces are included, momentum is conserved. In this sense, momentum conservation is one of the most fundamental properties of the universe.

All known fundamental interactions conserve momentum. This can be connected to the fact that space is the same everywhere, with the same laws of physics.

Collisions

The impulses on two colliding objects are equal and opposite:

$\Delta p_1 = -\Delta p_2$, $\quad$ $F_{12}\,t = -F_{21}\,t$.

We recognize this as Newton's 3rd law. This is where it came from.

Or, turning it around, the conservation of momentum for an isolated system is a consequence of Newton's 3rd law: since all internal forces cancel exactly, so do all internal momentum changes.

Example

For example, a railroad car of mass M_1 = 1600 kg could travel at u = 2.4 m/s and strike a second railroad car of mass M_2 = 3200 kg.

If the cars hitch together, how fast do they travel together after hitching?

Example

Before joining, the first car was the only one moving, so the total momentum was

$$p_1 = M_1\, u$$

After joining, the total momentum is

$$p_2 = (M_1 + M_2)\, v$$

Then the final velocity of the hitched cars is

$$v = \frac{M_1\, u}{M_1 + M_2} = u/3 = 0.80 \text{ m/s}$$

Car and Truck

Used in 9:00 class only.

A small car has a head-on collision with a large truck and they stick together.

Which vehicle has the bigger change in momentum?

- Neither – their change in momenta are equal and opposite.

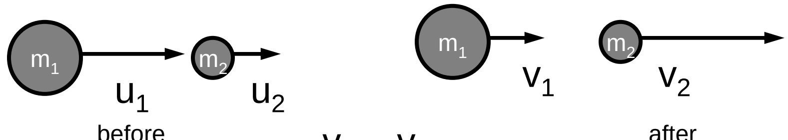
Which vehicle has the greater acceleration on impact?

- The force is the same (3rd law), so the car has more acceleration.

Elasticity of Collisions

When two objects stick together in a collision, we call it a (completely) inelastic collision. Collisions can have different degrees of stickiness or bounciness.

Compare the relative velocities before and after the collision. In a completely inelastic collision, the final relative velocity is zero. Normally, the final relative velocity might be some fraction or multiple of the original relative velocity, up to a minus sign.

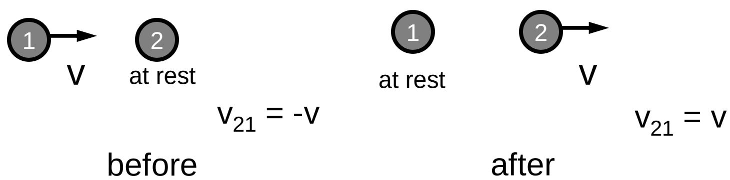


$$\frac{v_2 - v_1}{u_1 - u_2} = r$$

Elasticity of Collisions

The case where r = 1 is special: then the relative velocities just reverse direction in the collision. We call this an elastic collision. (In more than 1 dimension, the relative speeds are the same before and after, but not the direction.)

Example: Two equal masses, one at rest…



Note on the factor "r"

We won't be using the factor r much, but it has a name: the coefficient of restitution. (No need to remember this – the book doesn't use it.)

r = 0: inelastic.

r = 1: perfectly elastic.

0 < r < 1: typical bouncing, with dissipation.

r > 1? Yes – energy released in collision.

(bounce ball off mousetrap, throw ball while on ice, …)

Note on the factor "r"

added in response to a question that could have been answered better…

$r < 0$? This **is** possible if the objects can pass through one another.

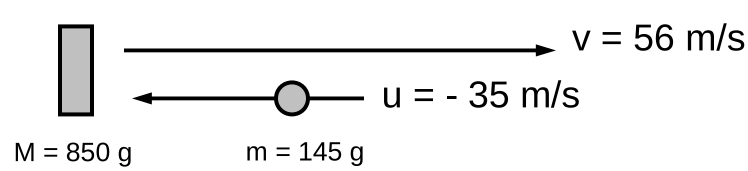
(Bullet through block of wood, …)

$-1 < 0 < -1$: objects pass, but with friction.

$r = -1$: $v_2 - v_1 = u_2 - u_1$, objects pass with no interaction.

$r < -1$: Objects give each other a push while passing.

Bat and Ball Example



Consider the example where a bat was swung at a ball pitched at 35 m/s, hitting the ball back on a line-drive at 56 m/s.

What was the speed of the bat before and after the collision if the bat has mass 850 g?

Bat and Ball Example

In general, we can't say, unless we know how "bouncy" the collision was. Let's assume the collision is elastic.

Write the initial and final speeds of the bat as U, V. (We'll use capital letters for the bat.)

Momentum conservation: $MU + mu = MV + mv$.

Elastic collision: $U - u = v - V$.

(Does the sign make sense?)

Bat and Ball Example

Put the bat variables on the left:

$$M(U - V) = m(v - u) = 13.2 \text{ kg m/s}.$$

Elastic collision: $U + V = u + v = 21$ m/s.

(Careful with signs!!)

Multiply the second equation by M = 0.850 kg and add:

$$MU - MV = 13.2 \text{ kg m/s}$$

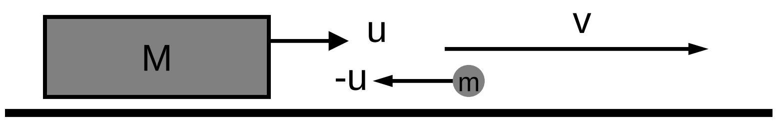
$$MU + MV = 17.9 \text{ kg m/s.}$$

$$(1.70\text{kg})\ U = 31.1 \text{ kg m/s.}$$

U = 18.3 m/s. V = 21 m/s – U = 2.7 m/s.

Train and Ball Example

Used in 10:00 class only.



Suppose a ball is thrown at a speeding locomotive, so that the ball and train both have initial speed u, but opposite directions.

What is the ball's speed v after the collision?

You can assume the ball is very light compared to the train, and collides elastically..

Train and Ball Example

Let's look at it from the point of view of the train.

The ball approaches at speed 2u. After the elastic collision, it leaves us at opposite speed 2u, relative to the train.

But the ball, being insignificant compared to the train's mass, does not affect the train's speed by a measurable amount.

So the train's speed is still u, and the speed of the ball relative to the ground is

$$u + 2u = 3u.$$

Force Acting Over Distance

When a constant force acts on a mass over a distance, the velocity changes.

Starting from rest,

$$F = m\,a = m\,(v^2/2x) = ½\,mv^2/x.$$

Then

$$½\,mv^2 = F\,x.$$

We call the left-hand side the **kinetic energy**, and the right-hand side the **work**.

Kinetic Energy and Work

- The kinetic energy is often written as $K = ½\,mv^2$.
- The units of energy are **Joules**:

$$1\ J = 1\ N\ m.$$

- Work is written $W = Fx$ (for constant force in 1 dimension).
- In general, $\Delta K = \int F dx = W$.
- More on work next week – now we concentrate on kinetic energy in collisions.

Energy Conservation

Momentum is conserved in an isolated system. What about energy?

If you define it properly, to include all types of energy, the total energy is conserved. This is as fundamental as momentum conservation, and related to the fact that the laws of physics are the same at all times.

Kinetic energy is not the only kind of energy, so it is not necessarily conserved by itself. Collisions that conserve kinetic energy are special.

Kinetic Energy In Collisions

How does the kinetic energy change in an elastic collision?

Consider the bat and ball. We had two equations:

$$MU - MV = mv - mu.$$

$$U+V = u + v.$$

Multiply (1) and (2):

$$M(U^2 - V^2) = m(u^2 - v^2).$$

Divide by 2 and rearrange:

$$\tfrac{1}{2} MU^2 + \tfrac{1}{2} mu^2 = \tfrac{1}{2} MV^2 + \tfrac{1}{2} mv^2.$$

Elastic collisions conserve energy.

Definition of Elastic Collision

We originally defined elastic collisions as ones in which the relative speed of the two objects doesn't change in the collision.

This is fine for two objects, but it is more general to define an **elastic collision** as one that **conserves energy**. This is independent of the number of objects colliding.

Problem Solving

- In 1d problems, the fact that the relative speed doesn't change ($v_2 – v_1 = u_1 – u_2$) is much more useful than energy conservation for solving problems.
- In 2d elastic collisions, energy conservation is often easier to use, since energy is not a vector. The relative velocity vector can change directions in an unspecified way in more than 1 dimension. More on 2d collisions Monday…

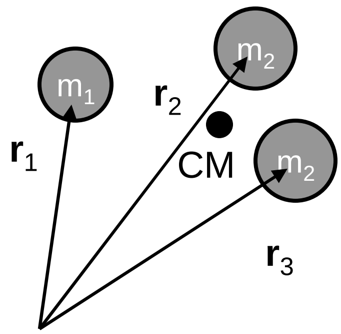
Center of Mass

We didn't get to these note's but I will leave them here as a supplement to the notes posted on Blackboard on the same topic.

In more dimensions you can use vectors:

$$\mathbf{r}_{CM} = \frac{\mathbf{r}_1 m_1 + \mathbf{r}_2 m_2 + \mathbf{r}_3 m_3}{m_1 + m_2 + m_3}$$

identifies the location in space where the three masses would balance in uniform gravity.



Center of Mass in Collisions

Writing the total mass as $M = m_1 + m_2$, the total momentum is

$$P = Mv_{cm}$$

Since P does not change in the collision, the CM velocity

$$v_{cm} = P/M$$

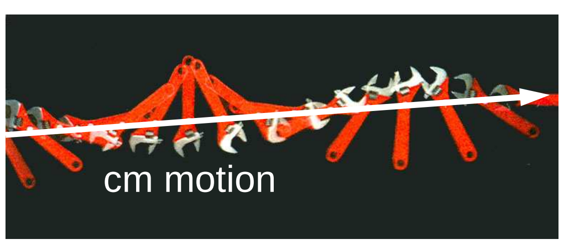
is constant.

Motion of Extended Objects

The motion of extended objects or collections of particles is such that the CM obeys Newton's 2nd Law.

Motion of the Center of Mass

The CM of a wrench sliding on a frictionless table will move in a straight line because there is no external force. In this sense, the wrench may be though of as a particle located at the CM.



Motion of the Center of Mass

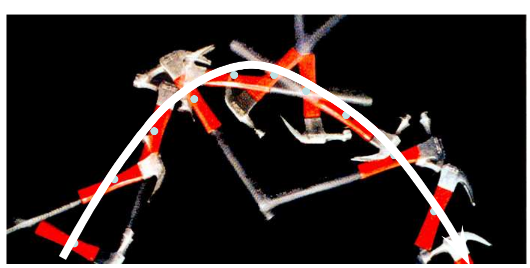
If an external force F acts on an extended object or collection of objects of mass M, the acceleration of the CM is given by

$$F = Ma_{cm}.$$

You can apply Newton's 2nd Law as if it were a particle located at the CM, as far as the collective motion is concerned. (This says nothing about the relative motion, rotation, etc., about the CM.)

Motion of the Center of Mass

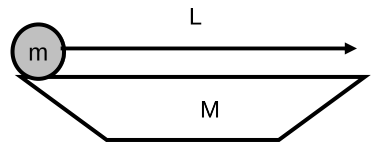
If a hammer is thrown, its CM follows a parabolic trajectory under the influence of gravity, as a point object would.



Person in Boat

A boat of mass M and length L is at rest in the water. A person of mass m walks from one end of the boat to the other. How far does the boat move?

Neglect friction (viscosity) between the boat and water.



Person in Boat

No external force implies fixed CM position.

$(M+m)x_{cm} = mx + MX$.

Initially, $x = 0$, $X = X_0$

Position of boat's CM.

After walk: $X = X_0 + D$.

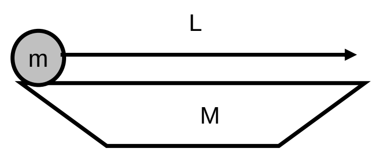
Boat moves a distance D.

$x = L + D$.

$MX_0 = m(L+D) + M(X_0 + D)$.

$(M+m)D = -mL$.

$D = -mL/(M+m)$.



Curious result beyond the scope of this question: If you don't neglect viscosity ($F = -kv$), the boat eventually goes back where it started!