Physics 103

Department of Physics
Princeton University

Precept Notes

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Circular Motion Part 1
Uniform Circular Motion

Announcements

- Read Knight Chapters 7-8.
- Problem set 3a is posted.
- The 105 admission quiz has been graded. If you took the quiz, you can decide if the course is appropriate for you based on your score and other factors.

Uniform Circular Motion

Position vector:

$R_x = R \cos \theta$, $R_y = R \sin \theta$

$\theta = \omega t$

ω = angular velocity,

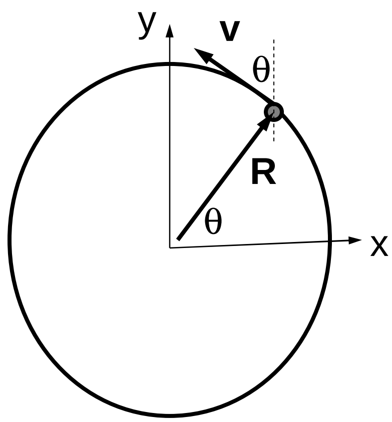
radians per second.

Period T for 1 revolution.

f = 1/T revolutions per second.

$\omega = 2\pi/T = 2\pi f$

Speed: $v = 2\pi R/T = R\omega$.



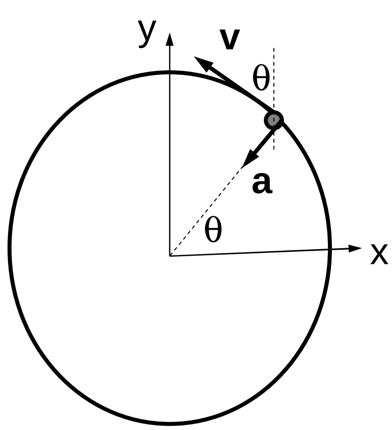
Uniform Circular Motion

Velocity vector:

$v_x = -v \sin \theta$, $v_y = v \cos \theta$

$\theta = \omega t$

Acceleration:
perpendicular to **v**, because speed is constant.



Uniform Circular Motion

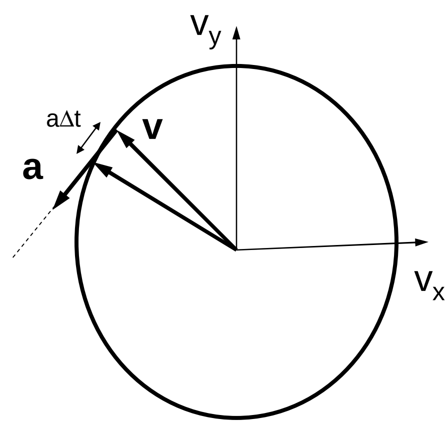
$a \, \Delta t = v\Delta\theta = v \, \omega\Delta t$

$a = v\omega$

$= v^2/R = R\omega^2$.

Vector: $\mathbf{a} = -\mathbf{R}\omega^2$

Since a is constant, you can also notice that the vector v traces out a circle of length $2\pi v$ in time T (period). The rate of tracing the circle is $a = 2\pi v/T = v\omega$.

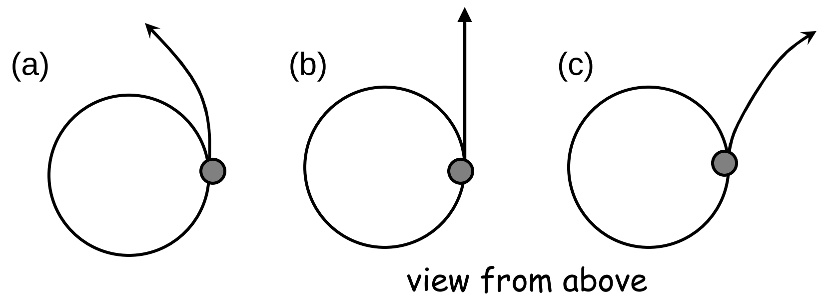


Centripetal Force

- From the fact that there is acceleration directed toward the center of a circle in uniform circular motion, we can infer that the net force on the object is **F** = m**a** also directed toward the center of the circle. This is called the **centripetal force**.

Circular Motion

If you twirl an object around your head on a string and then let go, which way does it travel?



Circular Motion

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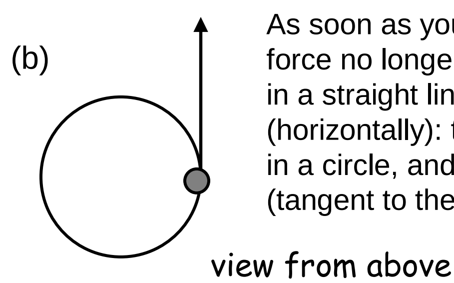


As soon as you let go, the centripetal force no longer acts, so the object moves in a straight line at a constant speed (horizontally): the same speed it was going in a circle, and in the same initial direction (tangent to the circle).

Noninertial Frames

When you travel in a car going around a curve, you “feel” a force pushing you outward.

However, you are in a non-inertial frame of reference in this case: forces felt in a non-inertial frame are not necessarily real.
- fictitious force.

Newton’s laws are valid in an inertial frame of reference, not an accelerated frame!

Noninertial Frames

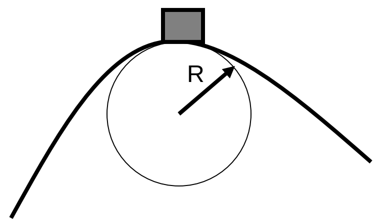
We will limit our discussion to inertial frames, so that all forces are real.

Don’t treat acceleration terms as forces in your diagrams.

Centripetal force is not some extra force due to circular motion, it is the resultant force of whatever is causing the circular motion!

Roller Coaster

A roller coaster car goes over a hill with radius of curvature R at speed v. How fast must it go for the riders to feel weightless at the top?



Note on General Motion

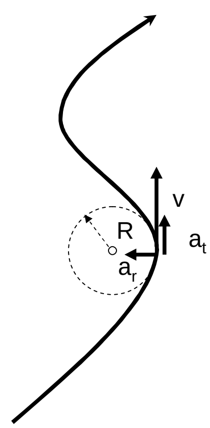
If an object is moving on any path at speed v(t), the acceleration can always be separated into a tangential term,

$$a_t = dv/dt,$$

and a radial term,

$$a_r = v^2/R$$

with R(t) the instantaneous radius of curvature of the path.



Roller Coaster

Recall that weightlessness means there is no contact force between the rider and car: $F_N = 0$.

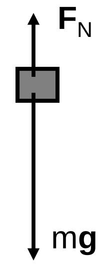
What vertical force remains?

- Gravity must be responsible for the full centripetal acceleration:

  $mg = mv^2/R.$

  $g = v^2/R.$

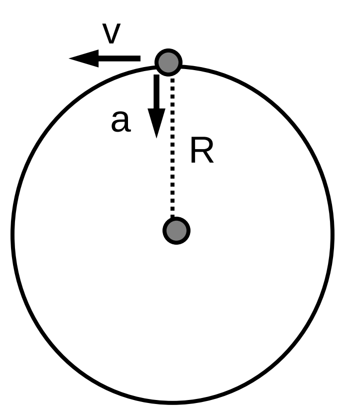
- The vertical acceleration of the rider due to the instantaneous circular motion on the track is identical to free-fall.



The Moon

Newton realized that gravitation is also the cause of the centripetal acceleration of the moon toward the earth:

$a = v^2/R$



The Moon

The velocity is related to the Moon's period.

$$v = 2\pi R / T$$

Therefore,

$$a = 4\pi^2 R/T^2$$

For the moon,

$R = 3.84 \times 10^8$ m, $T = 2.36 \times 10^6$ s

Therefore,

$$a = 4\pi^2 R/T^2 = 2.72 \times 10^{-3} \text{ m/s}^2$$

The Moon

This is much smaller than the acceleration of gravity on earth. In fact, $a = g/3600$

Newton compared this to the fact that the moon is 60 earth radii from the center of the Earth and noted that

$$ma/mg = a/g = (R_e/R)^2$$

The force of gravity is inversely proportional to the distance from the center of the earth.

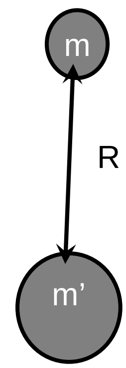
From this, Newton inferred the inverse square law of gravity: F_g is proportional to $1/R^2$.

The Moon

To infer a universal gravitational law from this, we can note that if $ma = C/R^2$ and all things fall at a constant rate, the strength of gravity has to be proportional to m, so C is proportional to m. The other object being attracted (Earth in this case) also has $m'a' = C/R^2$. So C is proportional to both masses: $C = Gmm'$.

Newton's law of gravity, $F_g = Gmm'/R^2$.

Measured value: $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$



Earth Orbit

What is the speed of a satellite orbiting two earth radii above the Earth's surface?

$(R = 3R_e)$

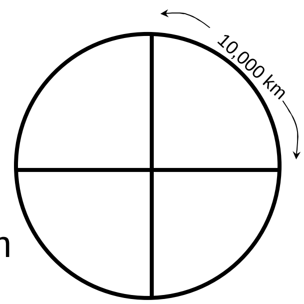
The inverse square law implies gravitational acceleration $a = g/9$.

Then $g/9 = v^2/3R_e$ with $R_e = 6380$ km.

$v = [(9.8\ m/s^2)(6.38 \times 10^6\ m)/3]^{1/2}$
$= 4.56$ km/s.

Size of Earth

The km was originally defined to be 1/10,000 of the distance from the equator to the north pole.



Then $2\pi R_e = 4\text{x}10^4$ km, giving

$$R_e = (2/\pi)\text{x}10^4 \text{ km} = 6370 \text{ km}$$

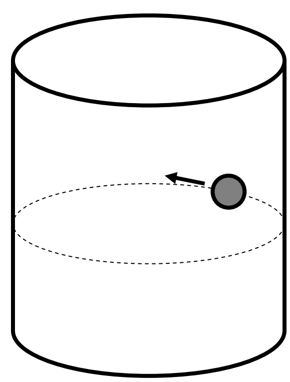
within 0.2% of the accepted value.

(The earth is not a perfect sphere.)

Knowing this can save looking it up.

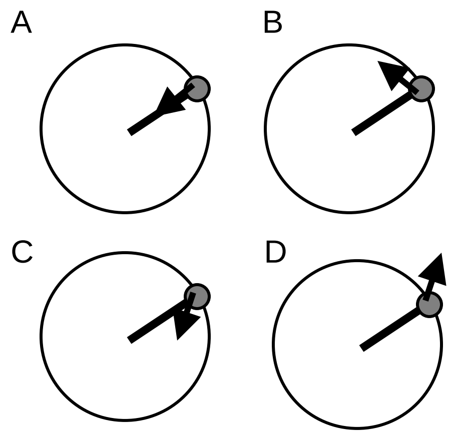
Trick Cyclist

A trick cyclist rides around the inside of a vertical cylinder. If the coefficient of friction of the tire on the cylinder is $\mu_s = 0.85$ and the radius of the cylinder is 5.0 m, what is the cyclist's minimum speed to avoid slipping down?



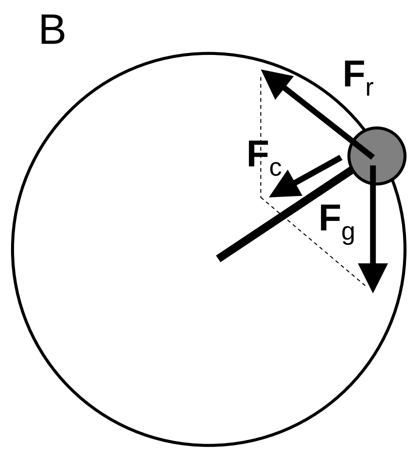
Mass on Rod

- A mass is rotated at constant speed in a vertical circle on the end of the rod. Which vector could possibly correctly show the force of the rod on the mass?



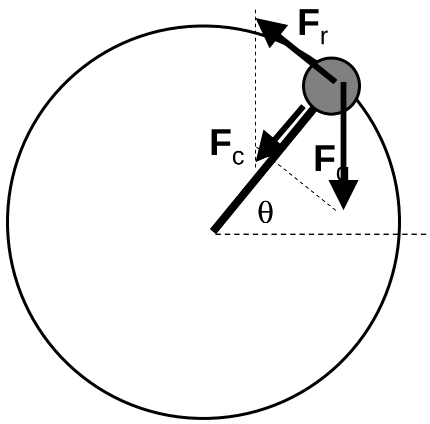
Mass on Rod

- The force of gravity and the force of the rod must add up to the net centripetal force, which acts toward the center in uniform circular motion.
- In general, $\mathbf{F}_r$ could be anywhere between the vertical direction and the rod's inward direction depending on the speed of the mass.



Mass on Rod

Find the components of $\mathbf{F}_r$ in terms of m, g , ω, and θ.



$F_c = mR\omega^2$

$F_{cx} = -\ mR\omega^2\cos\theta$

$F_{cy} = -\ mR\omega^2\sin\theta$

Mass on Rod

x component:

$F_{rx} = F_{cx}$

$= -\ mR\omega^2\cos\theta$

y component:

$F_{cy} = F_{ry} - mg$

$F_{ry} = F_{cy} + mg$

$= m(g - R\omega^2\sin\theta)$