Physics 103

Department of Physics
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Precept Notes

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Kinematics – Part 2

Accelerated Motion, Relative Motion

Announcements

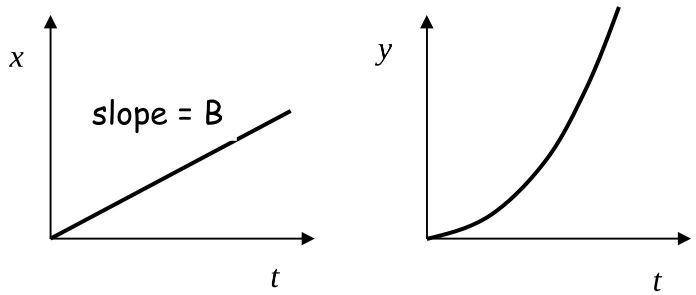
- Read Knight Chapters 4, 5 and 6.

  The topic is point particle dynamics – the study of motion of objects whose position matters, but not orientation.

  Note that chapter 5 was missing in the syllabus, but we will definitely be covering it!
- Problem set 2a is posted.
  - You do not have to read chapter 8 to solve the last problem. It is an F=ma problem.

Velocity Components

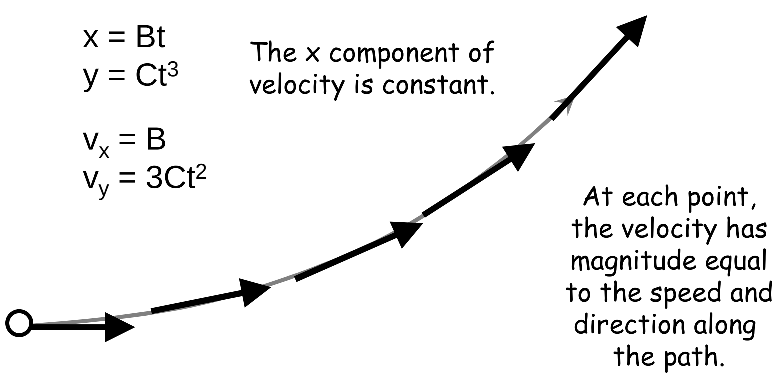
- Suppose x(t) = Bt and y(t) = Ct^3. Find **v**(t).



2 Dimensional Motion

x = Bt
y = Ct^3

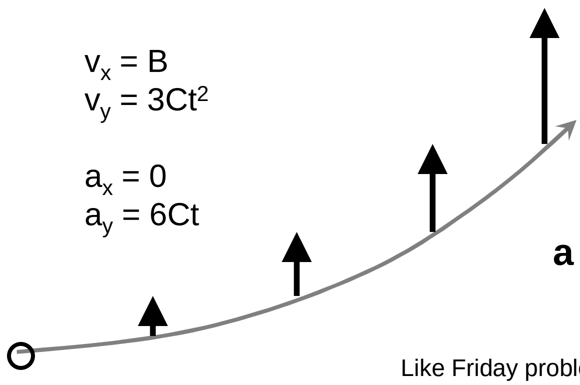
$v_x = B$
$v_y = 3Ct^2$



2 Dimensional Motion

$v_x = B$
$v_y = 3Ct^2$

$a_x = 0$
$a_y = 6Ct$

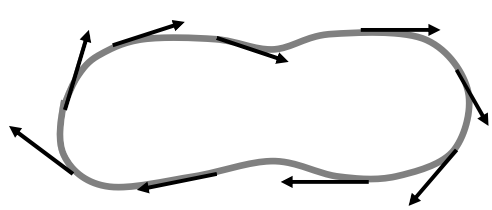


$\mathbf{a} = 6Ct\ \mathbf{j}$

Like Friday problem with "constant jerk" in the y direction.

Example

- A car drives along a track with the shape shown at constant speed. Draw the velocity vector and acceleration vector at enough points to show the general pattern.



Velocity vector:
Tangent to track,
Constant magnitude.

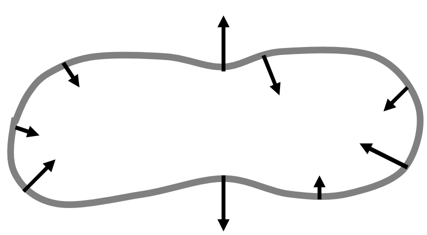
Question

- Can an object with a constant speed have a nonzero acceleration?

- *Yes* – if the speed is constant, the magnitude of the velocity is constant, but the direction changes.
- The acceleration is zero only if the velocity vector is constant.

Example

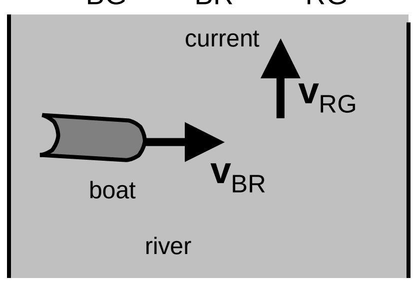
- Since the velocity component along the path is constant, the acceleration is always perpendicular to the path, showing which way it turns.
- The sharper the turn, the bigger the acceleration.

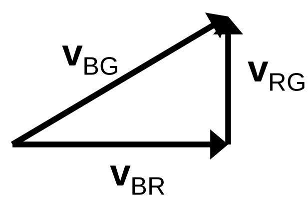


Acceleration vector.
Where is **a** = 0?

Relative Velocity

- If the river is flowing with velocity $\mathbf{v}_{RG}$ with respect to the ground ($\mathbf{v}_{RG}$ is in the direction in the picture) then the velocity of the boat with respect to the ground is $\mathbf{v}_{BG}$ $\mathbf{v}_{BG} = \mathbf{v}_{BR} + \mathbf{v}_{RG}$.





Relative Velocity

- **In general,** if object A moves with velocity $\mathbf{v}_{AB}$ from the point of view of object B, and object B moves with velocity $\mathbf{v}_{BC}$ from the point of view of object C, then object A moves with velocity $\mathbf{v}_{AC}$ from the point of view of object C, where $\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC}$.

Frames of Reference

The boat's velocity could be measured either by someone on the shore: $\mathbf{v}_{BG}$, or by someone floating in the water without paddling: $\mathbf{v}_{BR}$. The speed v_{BR} is often given as the boat's speed in still water, since that would be the same. The direction of $\mathbf{v}_{BR}$ is the way the prow of the boat points. The direction of $\mathbf{v}_{BG}$ is the actual direction the boat travels, and v_{BG} tells you how fast it gets there.

Either observer makes a valid measurement of the boat's velocity. They are said to be viewing the boat's motion from their **frame of reference**.

Galileo was first to realize that the natural state of motion of an object not interacting with anything was to have **constant velocity** – not to come to rest.

Frames of Reference

Frames moving at constant velocity play a special role in Newtonian physics, but none is more special than another. We need a way to transform between different reference frames. This is called a **Galilean Transformation**.

In words, the previous equation says that the difference between velocities of object C measured in reference frames B and A is the relative velocity of frame A with respect to frame B.

Relativity

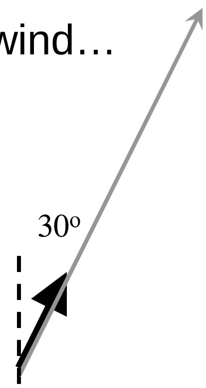
- The Galilean transformation turns out to be special because all observers moving at a constant velocity see the same laws of physics – the same forces and accelerations.
- But as intuitive as it seems, it is not the last word. At some point, physics had to come to terms with the fact that the 19th century laws of electricity & magnetism were *not* unchanged by a Galilean transformation. It is an observed fact that the speed of light turns out to be the same to all observers, no matter how fast they move! A modified transformation, the "Lorentz transformation", was constructed to implement it. This was an important ingredient in Einstein's reformulation of the structure of space-time in special relativity.

Plane in Wind

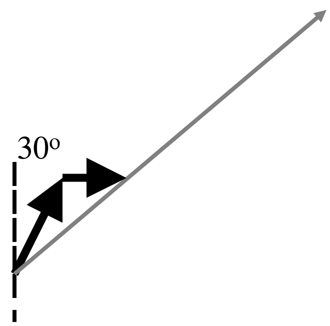
- A plane whose air speed is 180 km/hr heads at 30^o relative to N (compass heading), and is blowing at 70 km/hr from the west. What is the resulting velocity (magnitude and direction) with respect to the ground?
- Is 30^o the angle of the plane's velocity with respect to the ground or the air?

Plane in Wind

- No wind…

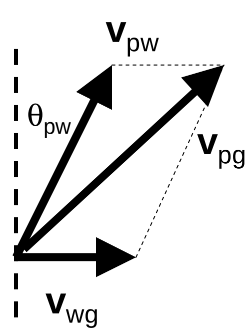


- Wind…



Plane in Wind

- The velocity of the plane relative to the ground is $\mathbf{v}_{pg} = \mathbf{v}_{pw} + \mathbf{v}_{wg}$.
- $v_{pg}^{x} = v_{pw} \sin \theta_{pw} + v_{wg}$
- $= 90$ km/hr + 70 km/hr
- $= 160$ km/hr.
- $v_{pg}^{y} = v_{pw} \cos \theta_{pw} = 156$ km/hr.

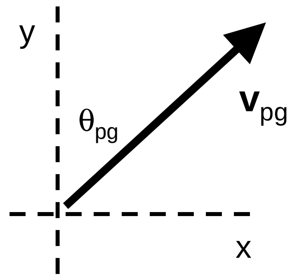


Plane in Wind

$$v_{pg} = \sqrt{(v_{pg}^x)^2 + (v_{pg}^y)^2}$$
$$= \sqrt{(160)^2 + (156)^2} \text{ km/hr}$$
$$= 223 \text{ km/hr}$$

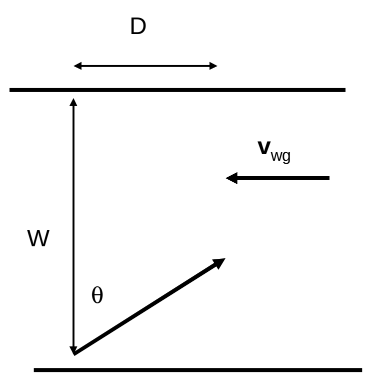
Relative to north:

$$\theta_{pg} = \text{atan}(v_{pg}^x/v_{pg}^y)$$
$$= \text{atan}(1.03) = 46^o$$



Boat Crossing River

- A boat whose speed in still water is 2.6 m/s must cross a river of width W = 280 m and arrive at a point D = 120 m upstream of where it starts. To do so the pilot must head the boat at $\theta = 48^o$. What is the speed of the river's current?



Boat Crossing River

- With respect to water, $v_{bw}^x = v_{bw} \sin\theta$, $v_{bw}^y = v_{bw} \cos\theta$, $v_{bw} = 2.6$ m/s.
- Current: $v_{wg}^x = v_{wg}$, $v_{wg}^y = 0$.
  (The sign convention is taken so the current shown is negative.)
- With respect to land:
  $v_{bg}^x = v_{bw} \sin\theta + v_{wg}$, $v_{bg}^y = v_{bw} \cos\theta$

How to use the information about distances?

Boat Crossing River

- Note: distance traveled in each direction is proportional to the component of velocity in that direction.
- $D/W = v_{bg}^x / v_{bg}^y$
  $= (v_{bw} \sin\theta + v_{wg}) / v_{bw} \cos\theta$.

Solve for vwg:

$v_{wg} = v_{bw} [(D/W) \cos\theta - \sin\theta]$

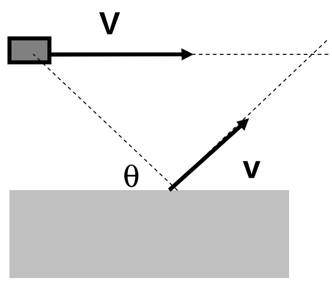
$= (2.6 \text{ m/s})[(0.42857)\ 0.669 - 0.743\ ]$

$= -1.19$ m/s (The sign means the current flows left.)

Marine Interception

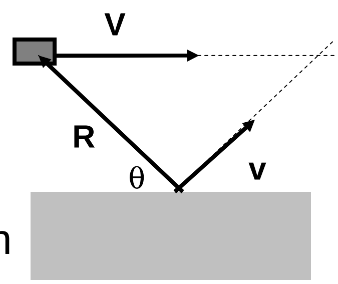
This example may not have been used in your class, but is a nice example of a problem that can be solved mostly using geometry rather than algebra.

- A ship steaming parallel to the coast at speed V is spotted by the coast guard, who launches a cutter from shore at speed v < V to intercept the ship. Assuming both ships maintain a constant velocity, what is the greatest angle θ the ship can have when the coast guard boat leaves port to be able to catch the ship?



Marine Interception

Think – what does it mean to be on a collision course in terms of the initial position vector **R** of the ship and the relative velocity $\mathbf{V}_{rel} = \mathbf{V} - \mathbf{v}$ of the ship with respect to the cutter?

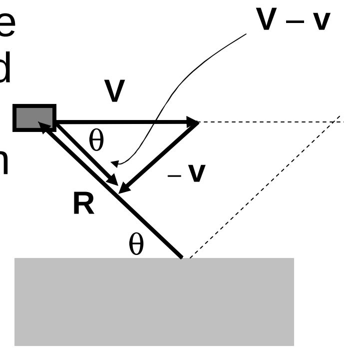


Marine Interception

There will be a collision if the ship appears to be headed toward you on the coast guard cutter at all times. In vector language,

R – $\mathbf{V}_{rel}$ t = 0
at some time t.

This means **V** – **v** is anti-parallel to **R**.

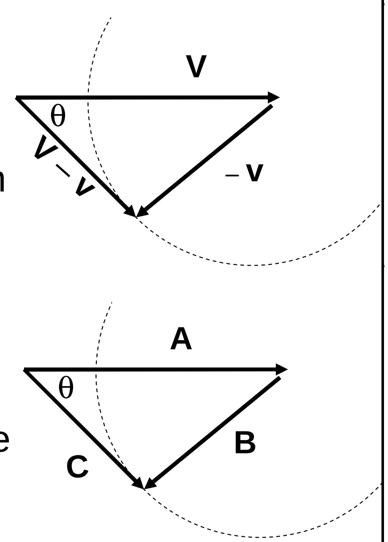


Marine Interception

The problem becomes an example we saw in the last precept: What is the maximum angle θ between **A** and **C** = **A** + **B** if the direction of **B** can vary?

Here, **A** = **V**, **B** = – **v**, and **C** = **V** – **v**.

C must be tangential to the circle swept out by **B**: So **B** ⊥ **C**.



Marine Interception

In the present problem, this implies that

$$\sin \theta = v/V.$$

The coast guard cutter needs to leave the shore while $\theta < \text{asin}\,(v/V)$.