Physics 103

Department of Physics
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Precept Notes

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Kinematics – Part 1
Vectors, Velocity, Acceleration

Note on these Slides

- These slides are made from my class notes with minimal editing. Since they are not intended for presentation, they are not meant to be readable on their own, but are provided as a convenience for those who would like a reminder of what was done in class.
- I am teaching two precepts: the exact content of each precept may vary. They are not meant to be lectures, and the material discussed may be partly in response to events in class. Some of this may not appear in these notes, or may differ for the two precepts. These slides are a partial hybrid of the notes used in the two precepts.

Announcements

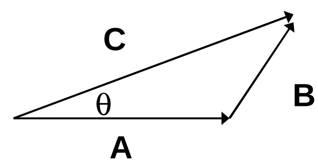
- Read Knight Chapters 1-3 and sec. 6.4.
  The topic is kinematics – the study of motion without regard to its cause.
- Three problem sets are posted:
  – Set 0 is not graded – but recommended
  – Set 1a is due tomorrow night, should be easy
  – Set 1b is due Monday night, a little harder
- If you want to be in 105, do the problems posted on the 105 web site: follow the link in the BlackBoard announcement, and see the syllabus for links to problem sets.

WebAssign Issues

- If WebAssign expects a symbolic answer, you must type it in a way it understands.
  – $\sin\theta$ must be written sin(theta)
  – Case matters: theta ≠ Theta
- Equivalent expressions are ok:
  – sin(theta) = cos(0.5*pi – theta).
- WebAssign will do simple calculations if it expects a number:
  – 6 = 3*2 = 2 + 4 = …

Vectors

- Vectors are perhaps the main mathematical construct we'll be using in kinematics. [Calculus is another.]
- They can be added geometrically or algebraically.



$C_x = A_x + B_x$

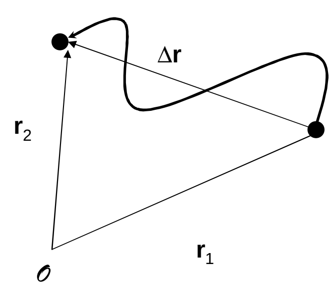
$C_y = A_y + B_y$

…

Vectors

- Vectors appear throughout kinematics:

  Position, Displacement, Velocity, Acceleration.



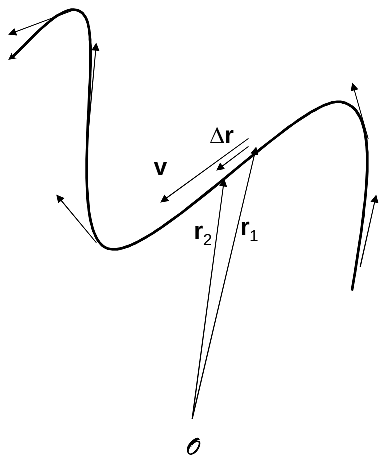
$\mathbf{v}_{avg} = \Delta\mathbf{r} / t$

$\mathbf{v} = d\mathbf{r}/dt$

$\mathbf{a}_{avg} = \Delta\mathbf{v} / t$

$\mathbf{a} = d\mathbf{v}/dt$

Velocity Vector



v = d**r**/dt = limit of
Δ**r**/Δt for small Δt.

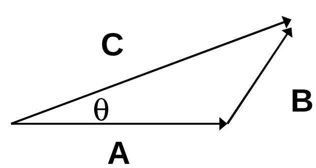
The velocity vector always points in the direction of motion and its magnitude is the **speed**.

In components:

$v_x = dx/dt$, $v_y = dy/dt$, …

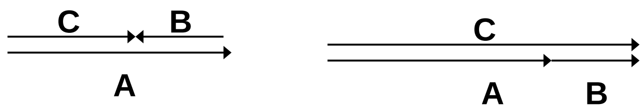
Vector Example

C = **A** + **B** with **A** pointing to the right and **B** pointing in an arbitrary direction you can vary. **A** has length 5 and **B** has length 3.



- What are the maximum and minimum magnitudes of **C**?
- What are the minimum and maximum angles θ between vectors **A** and **C**?

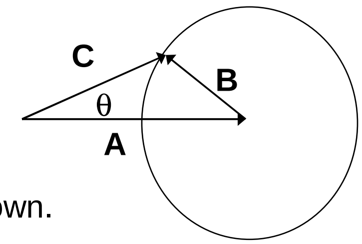
Vector Example



When **A** and **B** are *anti*-parallel, **C** is as short as possible, and has length C = 5 – 3 = 2.

When **A** and **B** are parallel, **C** is as long as possible, and has length C = 5 + 3 = 8.

Vector Example

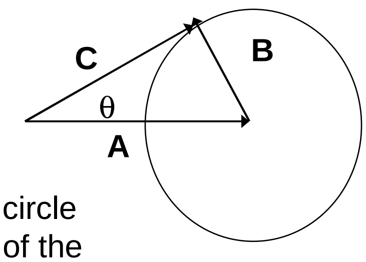


The vector **B** could point anywhere along the white circle shown.

The angle is clearly 0 when **A**, and **C** are parallel. The two cases on the previous slide both have θ = 0.

At what point along the circle will the angle θ become the largest?

Vector Example



The angle θ will be maximum when the vector **C** is tangent to the circle swept out by the possible locations of the tip of vector **B**.

Then **A**, **B** and **C** form the sides of a right triangle with hypotenuse **A**.

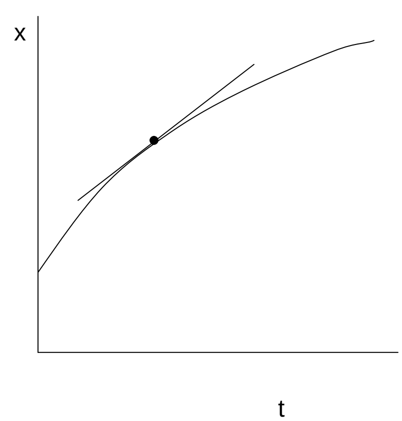
$\sin\theta = B/A = 3/5$, $\theta \approx 37^o$

Position-Time Diagrams

- A useful way to represent 1d motion is a space-time diagram. Draw the function x(t).
- What is the instantaneous velocity on this graph?

  [slope]
- Is the acceleration positive, negative, or both in different places?

  [negative]

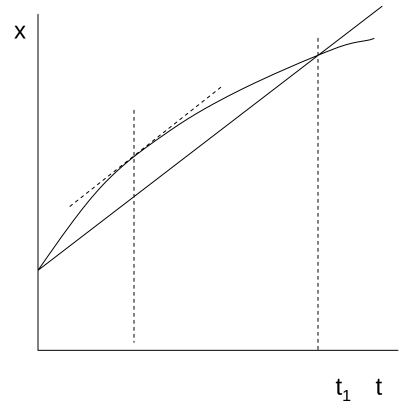


Position-Time Diagrams

- Trains A and B move as shown.
- At t_1, which is going faster?

  [A]
- Is there a time where they have the same speed?

  [yes]



Velocity and Acceleration

- Average velocity = displacement / time
- Average speed = distance traveled / time
- Instantaneous velocity v = dx/dt

  or more generally, $\mathbf{v} = d\mathbf{r}/dt$
- Average acceleration = $\Delta\mathbf{v}/\Delta t$.
- Instantaneous accleration $\mathbf{a} = d\mathbf{v}/dt$
- Constant acceleration:

  $x = v_0 t + ½ at^2$, $v = v_0 + at$,

  $v^2 – v_0^2 = 2a(x – x_0)$.

Average Speed

- Two cars go from Dallas to Austin and back. Car 1 goes 70 mph both ways. Car 2 pulls a trailer to Austin and can only go 60 mph, but drops it off and drives 80 mph back to Dallas. Which car makes the round trip faster?
- The one with the greatest average speed will be faster. Which has the greater average speed?
- Car 2 spends more time going 60 than going 80, since the distances are equal. The average speed is a time average, so the average will be closer to 60 than 80. Car 1 is therefore faster.

Average Speed

Find average speed of car 2…

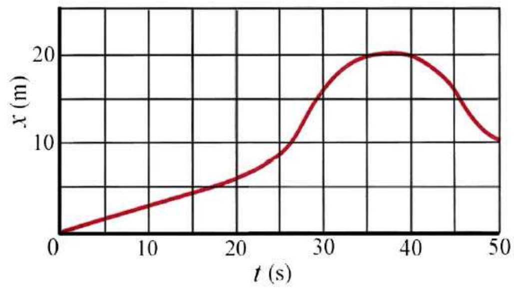
- $t_1 = D/v_1$, $t_2 = D/v_2$
- $t_1 + t_2 = D/v_1 + D/v_2 = D(v_1+v_2)/v_1v_2$

$v_{av} = 2D/(t_1 + t_2) = 2v_1v_2/(v_1+v_2)$

$= [2(60)(80)/140]$ mph $= 68.6$ mph.

Rabbit in Pipe

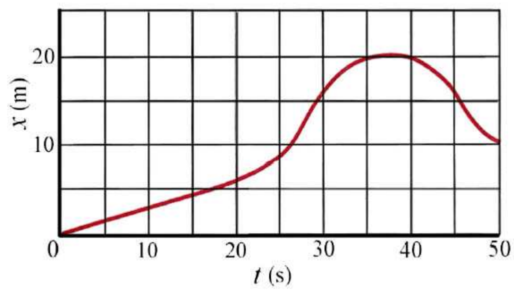
A rabbit runs through a pipe as shown.



When does the rabbit turn around? [37 s]
When is the speed the greatest? [28 s]
When is the acceleration the greatest? [21 s]

Rabbit in Pipe

What is the average velocity?
10 m / 50 s
= 0.2 m/s

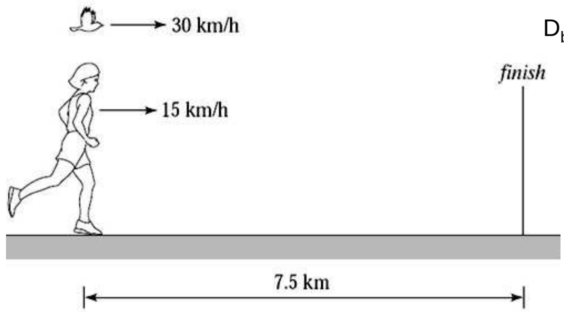


What is the average speed?

30 m / 50 s = 0.6 m/s

Runner and Bird

A runner runs at a steady pace of 15 km/hr. When she is 7.5 km from the finish, a bird begins flying from the runner to the finish at 30 km/hr. Every time the bird reaches the finish line, it turns around and flies back to the runner, repeating until the runner reaches the finish line. How far does the bird travel?



$D_{bird} = V_{bird}\, t_{runner}$

$= 30$ km (7.5 km/15 km)

$= 15$ km.

Toy Car

- A child's toy car rolling across a sloping floor is known to have constant acceleration. Taking x = 0 at t =0, it is observed that the car is at x=2.66 m at t = 1.0 s and x = 4.25 m at t = 2.0 s. What is the car's acceleration?

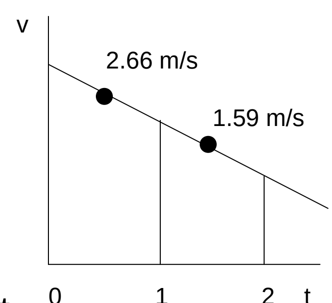
Toy Car

- If we know the positions and times, we can find the average velocities in the intervals from 0 – 1 s and from 1 – 2 s:

  v_{01} = 2.66 m/s, v_{12} = 1.59 m/s.
- How can this information be used to find the acceleration?

  If we know the instantaneous velocities at two points, we can calculate $a = \Delta v / \Delta t$.

Toy Car



- This is a constant acceleration problem with decreasing velocity, so the velocity decreases linearly with time.
- In a linear graph, the average over any interval is the value at the center of that interval.
- Thus, v(0.5 s) = v_{01} = 2.66 m/s,

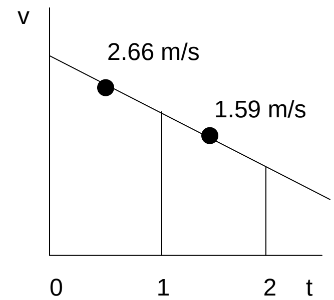
  v(1.5 s) = v_{12} = 1.59 m/s.

  a = (v(1.5 s) – v(0.5 s)) / 1 s = -1.07 m/s^2.

Toy Car

This slide was skipped in class.

- What is the car's initial velocity?



$v(0.5\ s) = v_0 + at,$

$v_0 = 2.66\ m/s$

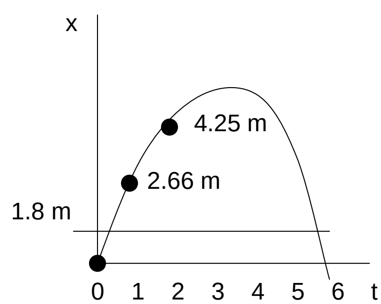
$- (-1.07\ m/s2)(.5\ s)$

$= 3.20\ m/s.$

Toy Car

This slide was skipped in class.

- When is the car at
  x = 1.8 m?



$x = v_0t + \frac{1}{2} at^2,$

Quadratic equation:

$\frac{1}{2} at^2 + v_0t - x = 0.$

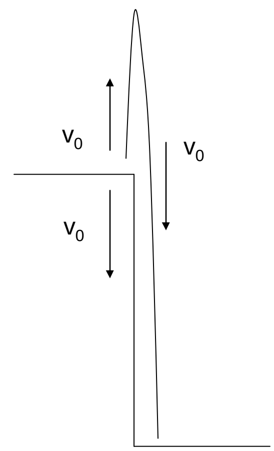
$0.535\ t^2 - 3.2\ t + 1.8 = 0.$

$$t = \frac{3.2 \pm ((3.2)^2 - 3.85)^{1/2}}{1.07} = 0.626\ s \text{ or } 5.35\ s.$$

Ball Thrown from Cliff

- Two people standing on the edge of a cliff throw balls with speed v_0.
- A throws the ball straight up, and B throws the ball straight down.
- Which ball lands with the greater speed? Neglect air resistance

[no difference]



Water Balloon

- A faculty resident of a dormitory sees a water-filled balloon fall vertically past his window. He observes that the balloon took 0.12 s to pass from the top of the window to the bottom, a distance of 1.8 m.
- Assuming the balloon was released from rest, how high above the top of his window was the guilty party?

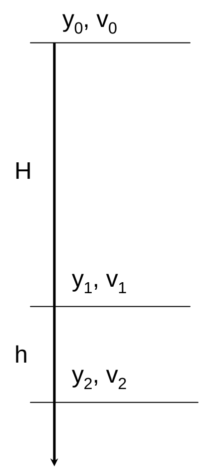
Water Balloon

- Draw a picture and label it.
- What do we know?

  t_{12} = 0.12 s,

  h = 1.8 m,

  g = 9.8 m/s^2.
- What do we need? H.



Water Balloon

Find some things we know:

- Average speed across window:

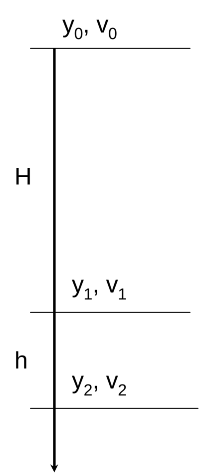
  v_{av} = h/t = 1.8 m / 0.12 s

  = 15 m/s.

  implying ½ (v_1+v_2) = 15 m/s.
- Constant acceleration:

  $v_2 - v_1$ = gt = 1.18 m/s.

Two relations between v_1 and v_2 mean we can solve for both if we want.



Water Balloon

Knowing v_1 would be useful:

$v_1^2 = 2gH$

So find it:

$v_1 + v_2 = 30$ m/s

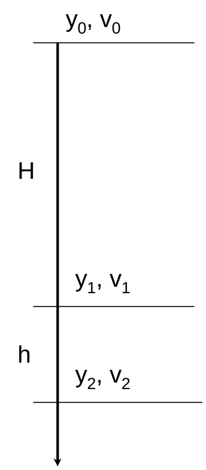
$v_2 – v_1 = 1.18$ m/s

Subtract:

$2\ v_1 = 28.8$ m/s.

$v_1 = 14.4$ m/s.

$H = v_1^2 / (2 \times 9.8\ \text{m/s}^2) = 10.6$ m.

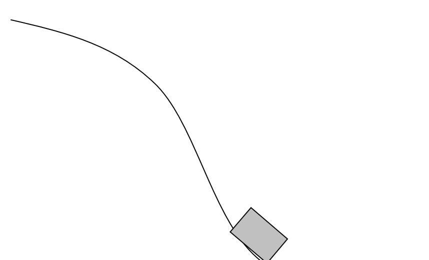


Roller Coaster

- If a roller coaster car is at the point shown, sliding only under the influence of gravity, is its speed increasing or decreasing?

  [increasing]

- Is its acceleration increasing or decreasing?



Roller coaster

- An object sliding vertically would accelerate at rate g.
- An object sliding horizontally doesn't accelerate, because gravity has no component in that direction.
- Infer: steeper slopes give more acceleration.

Roller Coaster

- The acceleration of the car is decreasing.
- The acceleration in the direction of motion is the component of g in that direction:

  g cos θ