3. Force and Gravity

Being in orbit is like being infatuated – you are constantly falling, but you aren't getting closer.

The Force of Gravity

Any two objects that have mass attract each other with a force we call gravity. You probably never noticed this for small objects, because the force is so weak. But the Earth has lots of mass, and so it exerts a big gravitational force on you. We call that force your *weight*. The fact that gravity is actually a force of attraction is not obvious. Prior to the work of Isaac Newton, it was assumed that gravity was simply the natural tendency of objects to move downward.

If you weigh 150 lb, and are sitting about 1 meter (3.3 feet) from another person of similar weight, then the gravitational force of attraction between the two of you is 10^{-7} lb. This seems small, but such forces can be measured; it is about the same as the weight of a flea.

You weigh less when you stand on the Moon, because the force of attraction is less. If you weigh 150 lb on the Earth, you would weigh only 25 lb on the Moon. You haven't changed (you are made up of the same atoms), but the force exerted on you is different. Physicists like to say that your *mass* hasn't changed, only your weight. Think of mass as the amount of material, and weight as the force of attraction of the Earth (or whatever other planet or satellite you are standing on).

Mass is commonly measured in kilograms. If you put a kilogram of material on the surface of the Earth, the pull of gravity will be a force of 2.2 lbs. So a good definition of a kilogram is an amount of material that weighs 2.2 lbs when placed on the surface of the Earth. That number is worth remembering.[1] Go to the surface of Jupiter, and you will weigh nearly 400 lbs. On the surface of the Sun you will weigh about 2 tons, at least for the brief moment before you are fried to a crisp. But in all cases your mass will be 68 kg.

The equation that describes the pull of gravity between two objects was discovered by Isaac Newton. It says that the force of attraction is proportional to the mass – double the mass and the force doubles. The force also depends on the distance. It is an *inverse square* law. It is inverse because when the distance gets larger, the force gets smaller. It is a square law because if you triple the distance, the force decreases by nine; if you make the distance increase by 4, then the force goes down by 16, etc.

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[1] A more accurate value is that there are 2.205 lb in a kilogram, and 0.4536 kg in a pound, but don't bother memorizing these more precise numbers.

The Equation of Newton's Law of Gravity

"Newton's Law of Gravity" gives the gravity force between two objects with masses *m* and *M* separated by distance *r*:

$$F = G\frac{m\,M}{r^2}$$

G is called the Gravitational Constant, and has the value $6.67\times10^{-11}\,N\cdot m^2kg^{-2}$ (N is for *Newton*, the physicists' unit of force) or $1.5\times10^{-11}\,lb\cdot m^2kg^{-2}$.
Let's go back to the example given in the text: two 150 lb people separated by 1 meter. The mass of each person is $\frac{150\,lb}{2.2\,\frac{lb}{kg}} = 68\,kg$. Putting these into the formula gives

$F = 1.5\times10^{-11}\,lb\cdot m^2kg^{-2}\times\frac{(68\,kg)^2}{(1\,m)^2}\approx10^{-7}\,lb$.

Newton's Law of Gravity actually gives the force only between two *small* objects. If one of the objects is a sphere (such as the Earth) then it turns out that you can still use the formula, but you must use the distance to the center of the sphere as the value for *r*. As an example, let's put in numbers for a 1 kg object sitting on the surface of the Earth. Then the force of attraction is given by the gravity equation with *m* = 1 kg, *M* = the mass of the Earth = 6×10^{24} kg, and *r* = radius of the Earth (that's the distance to the center of the sphere). This distance is r = 6371 km ≈ 6×10^{6} meters. Without plugging in the numbers, can you guess what the answer will turn out to be? Guess, and then check this footnote[2] to see if you guessed correctly.

Suppose you weigh 150 lbs on the Earth. Then your mass is $\frac{150\,lb}{2.2\,\frac{lb}{kg}} = 68\,kg$. What will you weigh on the Moon? We can calculate that by using Newton's Law of Gravity, and putting in the *M* = the mass of the Moon = 7.3×10^{22} kg, *r* = the radius of the moon = 1.7×10^{6} meters. The answer is *F* = 25 lb. That means you will weigh 25 lb. on the surface of the Moon.

Newton's Third Law

Here is something that might surprise you: if you weigh 150 lb, not only is the Earth attracting you with a force of 150 lb, but you are attracting the Earth with a force of 150 lb too. This is an example of something called "Newton's third law" – if an object exerts a force on you, then you exert the same force back on it.

[2] The answer is 2.2 lb. Of course, that is the weight of a 1 kg object.

Also, 1 newton ~ 4 ½ pounds, so we could express the answer as about ½ newton.

But if you are so small, how can you exert such a large force on the Earth? The answer is that, even though you are small, your mass exerts a force on every piece of the Earth, simultaneously. When you add all those forces together, the sum is 150 lb. So you are pulling up on the Earth exactly as much as the Earth is pulling down on you.

Think of it in the following way. If you push on some else's hand, they feel your force. But you feel the force too. You push on them; they push back on you. The same thing works with gravity. The Earth pulls on you; you pull on the Earth.

The "weightless astronaut" paradox

Imagine an astronaut in orbit in a capsule 200 km (125 miles) above the surface of the Earth. What does he weigh? Because he is further away from the mass of the Earth, the force is slightly lower. From Newton's Law of Gravity, we can calculate that he is weighs 142 lb, i.e. he is 8 lb lighter.

> **Calculation:**
> We can calculate the astronaut's weight by using Newton's Law of Gravity. Assume his mass is 68 kg. (That is his mass if he weighs 150 lb when standing on the surface.) The distance between him and the surface of the Earth is (6371 + 200) km (his altitude + the Earth's radius) = 6.57×10^6 meters. Plug that into the formula for the gravitational force and you get F = 142 lbs. On Earth he weighed 150 lbs. The astronaut weighs 8 lb less because he is further from the Earth. 200 km seems like a lot, but added to the radius of the Earth, it made only a 3% change in r, and that's why the weight didn't change by much.

But wait a minute – aren't orbiting astronauts weightless? Movies show them inside space ships floating around. How can they do that if they weigh almost as much as they do when they are on the surface of the Earth?

To understand the answer to this paradox, we have to think about what it means to be weightless. Suppose you are in an elevator, and the cable suddenly breaks. The elevator and you fall together. During those few seconds before you crash into the ground, you will feel weightless. You will float around inside the elevator. You will feel no force on your feet, and your shoulders will not feel the weight of your head. (Your head falls with your chest, at the same rate, so the muscles in your neck need exert no force to keep the head above the chest.) In those brief seconds you have the same "weightless" experience as the astronauts. All the time, of course, the Earth is pulling you rapidly towards it.[3] You have weight, but you feel weightless. A movie made of you inside the elevator would show you floating around, apparently without weight, while you and the elevator

[3] There are rides at some amusement parks that allow you to fall for long distances and experience weightlessness, at least for a small number of seconds. We'll calculate how many seconds in a later section.

fell together. You would look just like the astronauts floating around in the Space Station.

Now imagine that instead of falling, you and the elevator are shot together out of a gun, and fly 100 miles before hitting the ground. During that trip, you will again feel weightless. That's because you are in motion along with the elevator. You and it fly in the same arc.[4] Your head and chest are both moving in that arc together; there is no force between them and your neck muscles can be completely relaxed. Your head will seem to have no weight. Prior to the space program, potential astronauts were flown in airplanes following such arcs in order to see how they responded to the sensation of weightlessness – and to get them used to it.

When you and the elevator are moving together under the force of gravity (either falling or shot in an arc) there seems to be no gravity. From that alone, you might think you were far out in space, far away from the gravity of any planet, star, or moon. From inside the elevator, you can't tell the difference.

Now imagine that at the top of a very tall tower (200 km high) is a large gun, pointing horizontally. We shoot the elevator and you horizontally. If we picked a low velocity (e.g. 2 km/sec) you and the elevator would curve towards the Earth, and you would crash into it, as in path *A* in the figure below. But instead we pick a high velocity: 8 km/sec. You and the elevator are shot from the gun, and you curve towards the Earth, but because of your high velocity, you miss the edge of the Earth, as in path *B*. You keep on curving downward, but you never hit. You are in orbit. The force of gravity makes the path of the elevator – let's call it a space capsule now – curve downwards. But if that curvature matches the curvature of the Earth, then it misses the surface, and stays at a constant height.[5]

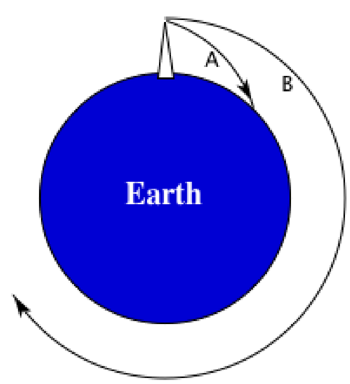

Figure: Capsule shot into space from a tower

[4] Your path is a geometric curve known as a parabola.

[5] If your velocity is not exactly horizontal, or if your velocity is a little low or high, then the orbit will not be a circle but an ellipse.

This may seem preposterous, but it is reasonable to think of an astronaut in orbit around the Earth as being in a state of perpetual falling. That's why he feels weightless.

You can think of the Moon as doing the same thing. It is attracted to the Earth by gravity, but it has high sideways motion. Even though it is falling towards the Earth, is always misses.

The Velocity for Low Earth Orbit (LEO)

To stay in a circular orbit just a few hundred miles above the surface, the velocity of the satellite must be about 8 km per second, which is about 18,000 miles per hour. (The actual value depends slightly on the altitude; we'll derive this number from a calculation later in this chapter.) At this velocity, the satellite orbits the 24,000 miles circumference of the Earth in about 1.5 hours.

If the astronaut wants to land, he does *not* point his rockets downward and fire his rockets away from the Earth; he fires his rockets towards the direction he is headed – in the forward direction! That slows down the satellite, so it is no longer going fast enough to miss the edge of the Earth. Gravity brings the satellite back to Earth. If the satellite moves faster than 8 km per second, it leaves the circular orbit and heads out into space. At about 11 km per second it will have sufficient velocity to reach the Moon and beyond. This velocity is called the escape velocity. We'll discuss this concept further later in this chapter.

Analogy with a Rock and Sling

There is another way to think about Earth satellites. Forget gravity for a moment. Imagine that you have a rock tied at the end of a string, and you are spinning it in a circle above your head. The string provides the force that keeps the rock from flying away, that keeps the rock in circular motion. If the string breaks, the rock flies off in a straight line. Gravity does the same thing for an Earth satellite: it provides the force that keeps the satellite in a circular orbit.

An old weapon called the "sling" is based on this principle. A rock is held by a leather strap, and spun in circles over the head. Arm motion helps it pick up circular speed. It is the strap that keeps the rock in circular motion. When the strap is released, the rock flies in a straight line towards its target. Such a sling was the weapon that, according to the Bible, David used to kill the giant Goliath.

In a similar manner, if we could suddenly "turn off" the force of gravity, the Moon would leave its circular orbit, and head off in a straight line. Likewise for all the satellites in orbit around the Earth. And with the Sun's gravity turned off, the Earth would head out into space too, at its previous orbital speed of 30 km/sec (67,500 mph).

Geosynchronous Satellites

Weather satellites and TV satellites have a very special orbit: they are "geosynchronous." This means that they stay above the same location of the Earth at all times. That means that the same weather satellite will be able to watch the development of a storm, or of a heat wave, continuously. It also means that if you are receiving a signal for your TV, you never have to re-point the antenna. The satellite remains in the same direction above your house at all times.

How can this be, since satellites must orbit the Earth to avoid crashing back down? The answer is elegant: the satellites orbit the Earth at such a high altitude (where the gravity is weak) that they go at a low velocity, and take 24 hours for each orbit. Since the Earth rotates once in that period, they stay above the same location. Both are moving – your home with the TV dish, and the satellite – but their angle with respect to each other doesn't change.

Geosynchronous satellites orbit the Earth at the very high altitude of 22,000 miles, over 5 times the radius of the Earth. Remember that the force between two objects had a $1/r^2$ in it? If the distance is 5 times larger, then that factor makes the force 25 times smaller. Moreover, at the high altitude the distance to make a circular orbit is longer. These factors combine to make the time to circle the Earth equal to 24 hours, rather than the 1.5 hours of a LEO.

There is a catch. If the satellite is to stay in the same location in our sky relative to the ground, it must orbit above the equator. Can you see why that is true? A geosynchronous satellite moves in a circle around the center of the Earth. If the satellite is not in an equatorial orbit, then it will spend half of its orbit in the Northern Hemisphere, and half in the Southern. Only if it orbits above the equator can it stay precisely above the same Earth location at all times.

As a result, all geosynchronous satellites are right above the equator. If you look at them up in the sky, they all line up in a narrow arc. In fact, there is so little room left, that international treaties are required to divide up the space. (If satellites are too close to each other, their radio signals can interfere.)

If you are kidnapped, and don't know where you have been taken, try to spot a satellite dish. If the dish is pointing straight up, then you know you are on the Equator. If it is pointing horizontally, then you know you are at the North pole.[6]

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[6] But be careful. Even at the equator, the satellite doesn't have to be overhead. The satellite could be above the Congo and you could be in Brazil. So you really have to determine the direction of north to make good use of the satellite dish information.

Spy Satellites

Spy satellites are satellites that carry telescopes to look down on the surface of the Earth and see what is going on. They were once used exclusively by the military, to try to see the secrets of adversaries, but now they are widely used by government and industry to look at everything from flooding and fires to the health of food crops.

The ideal spy satellite would stay above the same location all the time. But to do that, it must be geosynchronous, and that means that its altitude is 22,000 miles. At those distances, even the best telescopes can't see things smaller than about 200 meters. (We'll derive that number when we discuss light.) That means that such a spy satellite could see a football stadium, but couldn't tell if a game was being played. Such satellites are good enough to watch hurricanes and other weather phenomena, but are not useful for fine details, such as finding a particular terrorist.

Thus, to be useful, spy satellites must be much closer to the Earth. That means they must be in low earth orbit (LEO), no more than a few hundred miles above the surface. But if they are in LEO, then they are not geosynchronous. In LEO they zip around the 24,000 miles circumference of the Earth in 1.5 hours; that gives them a velocity relative to the surface of 16,000 miles per hour. At this velocity, they will be above a particular location (within ± 100 miles of it) for only about 7.5 minutes.[7] This is a very short time to spy. In fact, many countries that want to hide secret operations from the United States keep track of the positions of our spy satellites, and make sure their operations are covered over or hidden during the brief times that the spy satellite is close enough to take a photo.

LEO satellites cannot hover. If they lose their velocity, they fall to Earth. If you want to have continuous coverage of a particular location, you must use a circling airplane, balloon, or something else that can stay close to one location.

GPS – Medium Earth Orbit Satellites (MEO)

One of the wonders of the last decade is the GPS satellite system. GPS stands for "Global Positioning System", and if you buy a small GPS receiver (cost under $100), it will tell you your exact position on the Earth within a few meters. I've used such a receiver in the wilderness of Yosemite, in the souks of Fez, and in the deserts of Nevada. You can buy a car with a built-in GPS receiver that will automatically display a map on your dashboard showing precisely where you are. The military uses GPS to make its smart bombs land at just the location they want.

The GPS receiver picks up signals from several of the 24 orbiting GPS satellites. It is able to determine the distance to each satellite by measuring the time it took the signal to

[7] At 1600 miles per hour, it will go 200 miles in 1/8 of an hour = 7.5 minutes.

go from the satellite to the receiver. Once it knows the distance to three satellites, it can then calculate precisely where on Earth it is.

The GPS satellites were not put in geosynchronous orbit, because the great distance would require that their radio transmitters have much more power to reach the Earth. They were not put in low orbit (LEO) because they would then often be hidden from your receiver by the horizon. So they were placed in a medium Earth orbit (MEO) about 12,000 miles high. They orbit the Earth every 12 hours.

To understand how GPS works, consider the following puzzle. A person is in a U.S. city. He is 800 miles from New York City, 900 miles from New Orleans, and 2,200 miles from San Francisco. What city is he in?

Look on a map. There is only one city that has those distances, and that is Chicago. Knowing three distances uniquely locates the position. GPS works in a similar manner, but instead of measuring distances to cities, it measures distances to satellites. And even though the satellites are moving, their locations when they broadcast their signals are known, so the computer in your GPS receiver can calculate its position.

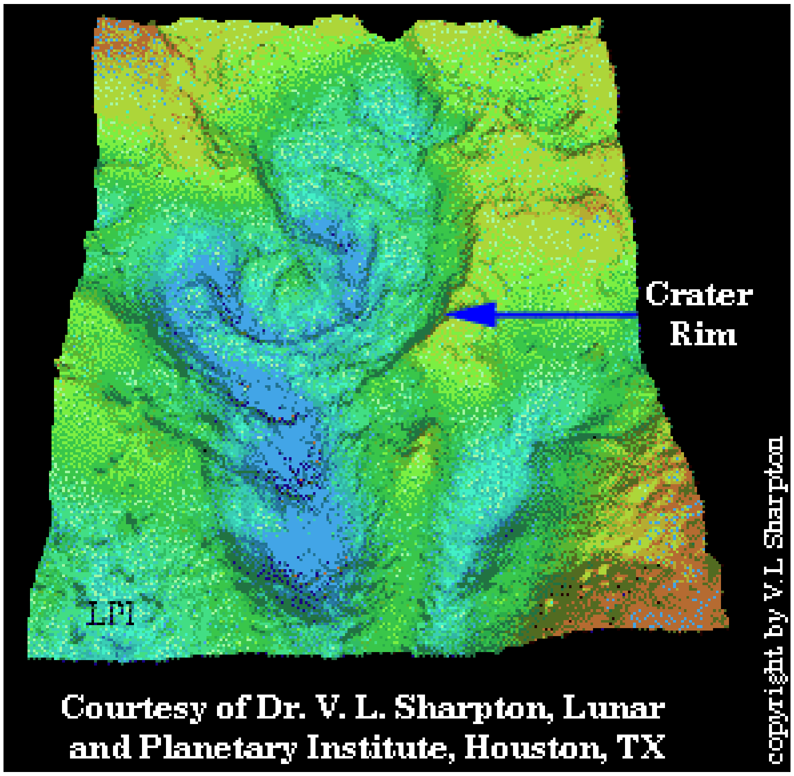
Using gravity to search for oil

It was said earlier that every object exerts a small gravitational force on every other object. Remarkably, measurement of such small forces has important practical applications. If you are standing over an oil field, the gravity you feel will be slightly less than if over solid rock, for the simple reason that oil weighs less, and so its gravity isn't as strong.[8] Such small gravity changes can even be measured from airplanes flying above the ground. An instrument can make a "gravity map" that shows the density of the material under the ground. Maps of the strength of gravity, taken by flying airplanes, are commonly used by businesses to search for oil and other natural resources.

A more surprising use of such gravity measurements was to make a map of the buried crater on the Yucatan peninsula, the crater left behind when an asteroid killed the dinosaurs. The crater was filled in by sedimentary rock that was lighter than the original rock, so even though it is filled, it shows a gravity "anomaly," i.e. a difference from what you would get if the rock were uniform. An airplane flying back and forth over this region made sensitive measurements of the strength of gravity, and they produced the map shown below. In this map, the tall regions are regions in which the gravity was

[8] The gravity of a spherical objects acts as if it is all coming from the center of the Earth. That is true only if the object's mass is uniformly distributed. If there is an oil field, then you can mathematically think of that as a sum of a uniform Earth, and a little bit of "negative" mass that cancels out some of the gravity. If you are close to the oil field, you will sense the reduced gravity because this little bit of negative mass will not attract you as much as if it were denser rock.

stronger than average, and the low regions were locations where the gravity was slightly weaker.



The crater shows several concentric circles, with the largest over 100 km in diameter. The inner rings probably formed when the huge crater initially filled in as material from under the crater was forced upward, partially filling it.

Manufacturing objects in a weightless environment

When the space program began, many people thought that there would be significant advantages to being in a weightless environment. In a satellite, things wouldn't sag under their own weight. It might be possible to make better (rounder) ball bearings, or to grow more perfect crystals (used in computers and other electronics) than can be done on the ground.

This promise has been largely unfulfilled. The additional cost of doing the work in a satellite has not turned out to be worth it. It costs about $10,000 to launch a kilogram of anything into orbit. In the near future, commercial companies hope to reduce that price to $1,000 per kilogram. It's hard to make a profitable factory in space when it costs that much just to get there. There is no reason in principle why getting to space *must* be expensive; we'll show in a later section that the energy required is only 15 Cal per gram. Some time in the future, if travel to space becomes as cheap as an airplane ride, then the idea of factories in orbit may be more likely.

Measuring the Mass of the Earth

Have you wondered how we know the mass of the Earth? You can't just put it on a scale. The first person to determine it was Henry Cavendish, in 1798. He did it in a very indirect way. Even though Newton had discovered the equation of gravity, at the time of Cavendish nobody knew the value of the constant *G*. Cavendish determined it by taking several masses in his laboratory and measuring the force of gravitational attraction between them. (It was not an easy experiment; as we said earlier, the force you get is about the same as the weight of a paramecium.) Once he knew *G*, he could figure out the mass of the Earth from the known distance to the moon, and the fact that it goes around once every 28 days. When asked what he was doing in his lab, his answer (some people claim) was "weighing the Earth!"

Problem: How could you use similar principles to determine the mass of the Sun?

Gravity on the Moon and Asteroids

Our Moon has about 1/81 of the mass of the Earth. So you might think its gravity would be 81 times less. But its radius is only 1/3.7 that of the Earth. Remember that gravity is an inverse square law, so from the small radius you would expect the force to be $(3.7)^2 = 13.7$ times larger. If you combine these two effects, you get the surface gravity is $\frac{13.7}{81} \approx \frac{1}{6}$ that on the Earth. And that is the value the astronauts found when they landed there. The gravity was so weak that they seemed to bounce around in slow motion. When they jumped, they went high, and when they came back down, they came down slowly.

What is the surface gravity of an asteroid, which has a radius of only 1 km? We can't say, since we don't know the mass. However, if you assume that the density is the same as for the Earth, then we can derive a simple result: the surface gravity is proportional to the radius.[9] Since the radius of the Earth is 6378 km, this means that the surface gravity on an asteroid is 1/6378 of the value on the Earth. If you weigh 150 lb on Earth, you would weigh 150 lb/6378 = 0.023 lb, or about a third of an ounce. That's about the weight of three pennies on the Earth.

As we'll show in a later section (on escape velocity), you have to be very careful, because your escape velocity would be very low. To jump into space from the Earth requires a velocity of 11 km/sec, but from the asteroid you would only require a velocity of 2 m/s.

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[9] That's because the mass of the asteroid is $M = \text{density} \times \text{volume}$. Call the density d. The volume of a sphere is $\frac{4}{3}\pi R^3$. So the gravity at the surface is

$$F = G\frac{Mm}{R^2} = Gm\frac{M}{R^2} = Gm\frac{\frac{4}{3}\pi R^3 d}{R^2} = \text{constants} \times R.$$

So the weight of a mass of m = 1 kg on different planets will depend only on their radius *R* of that planet (if all planets are of similar material composition).

(The same jump speed would get you less than a foot high on the surface of the Earth.) You could easily launch yourself into space by jumping. This low escape velocity was a problem for a U.S. space probe called "NEAR" (stands for Near Earth Asteroid Rendezvous). If the satellite landed at a velocity of 2 m/s or more, then it might have bounced right back out into space.

Gravity in Science Fiction

One of the most common "errors" in Science Fiction movies is the implicit assumption that all planets in all solar systems have gravity about equal to that on the Earth. There is no reason why that should be so. Pick a random planet, and you are just as likely to be a factor of 6 lighter (and bouncing around like astronauts on the Moon) or six times heavier and unable to move because of your limited strength. Imagine a person who weighs 150 lb on the Earth, trying to move on a planet where he weighs 900 lb.

> **Numerical exercise**:
> Use the simple radius formula to estimate the surface gravity on the moon. How close is your answer to the correct one that we calculated? Why aren't the answers the same? Can you guess how far wrong the approximate formula must have been when applied to the asteroid?

Falling to Earth

Let's now talk about everyday gravity, the gravity that you feel when you are near the surface of the Earth. When you jump off a diving board (or a bungee tower) gravity pulls you downwards. The force of gravity acts on all parts of your body, and makes them fall faster and faster. You accelerate, but in a very remarkable way: every second that you fall, you pick up an additional 9.8 meters per second of velocity. Put in the form of an equation, your velocity v after a time t is

$$v = gt$$

where the constant g = 9.8 m/s^2 is usually called the *acceleration of gravity*.

By *acceleration* we mean the rate at which your velocity changes. For gravity, the acceleration is constant. After one second, your velocity is 9.8 meters/sec. After two seconds, it is 19.6 m/s. After three seconds, 29.4 m/s. After 4 seconds, 39.2 m/s. Every second you pick up an additional 9.8 m/s.

Here is a useful conversion factor:

1 m/s = 2.24 mph.

After falling for 4 seconds, our velocity was 39.2 m/s. To convert this to mph, just multiply by 2.24. So the velocity after 4 seconds is $39.2\,\frac{m}{s} \times 2.24\,\frac{mph}{m/s} = 88\,\text{miles per hour}$.

That may give you a better sense of how fast you're moving. How far will you fall in this time? The equation can be derived using calculus.[10] The answer is:

$$D = \frac{1}{2} g t^2 .$$

You are not required to memorize this equation; it is shown so you can see how we do these calculations. If we put in $t = 4$ seconds, this gives[11] $D = 78$ meters (257 feet). That's how far you'll fall in 4 seconds.

We can also use the equations backwards to see how long it takes to fall a certain distance. In 1939, when King Kong fell off the Empire State Building, it was 330 meters high.[12] So how long should it have taken him to fall from the top to the bottom? Take our equation for *D* and solve for *t*. This gives:

$$t = \sqrt{\frac{2D}{g}} .$$

Put in $D = 330$ m, $g = 9.8$ m/s^2, to get that $t = 8.2$ seconds. Watch the movie (the 1939 version) and see if this matches what is in the movie.

How fast was Kong moving when he hit the ground? We just calculated that $t = 8.2$ seconds. We can put that into our velocity equation to get

$$v = gt = 9.8 \tfrac{m}{s^2} \times 8.2 s = 80 \tfrac{m}{s}$$

Remember that 1 m/s is 2.24 mph. That means that Kong was falling at $2.24 \tfrac{mph}{m/s} \times 80 \tfrac{m}{s} = 180 mph$ when he hit the ground. No wonder he was killed.

Dropping food, terminal velocity, and parachutes

In 2002, during the U.S. war in Afghanistan, food was dropped out of airplanes to feed the Afghan people. The food was dropped from an altitude of 3000 meters (about 10,000 ft). How fast was it going when it hit the ground?

Let's use the equations to figure this out. The time it took to fall was:

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[10] Optional footnote: According to calculus, the distance is $D = \int v\,dt = \int gt\,dt = g \int t\,dt = \tfrac{1}{2} g t^2$.

[11] $D = \tfrac{1}{2} g t^2 = \tfrac{1}{2}(9.81 \tfrac{m}{s^2}) \cdot (4s)^2 = 78m$

[12] In 1939 the Empire State Building was only 6 years old, and did not yet have the 250 foot TV antenna on its top.

$$t = \sqrt{\frac{2D}{g}} = \sqrt{\frac{2 \times 3000\, m}{9.8 \frac{m}{s^2}}} = 24.7\, s$$

and its velocity would have been

$$\begin{aligned} v = gt &= 9.8 \tfrac{m}{s^2} \times 24.7\, s = 242.5 \tfrac{m}{s} \\ &= 543.2\, mph \end{aligned}$$

Ouch! That would be deadly. However, we made the mistake of neglecting the force of air as the packages were dropped. Every time the package hits a molecule of air, it transfers some of its energy. The force *F* of air depends on the area *A* of the package (more area means it hits more air molecules). That's not surprising. But more interesting is the fact that the force depends on the square of your velocity. Double your velocity, and the force goes up four times; go 3 times faster, and your force increases by 9. This means that air resistance becomes extremely important at high velocities. This fact is the key to understanding not only dropped food, but also parachutes, space capsule reentry, and automobile gasoline efficiency.

> The air resistance equation is:
>
> $$F = \tfrac{1}{2} A \rho v^2$$
>
> The symbol ρ is the density of air, 1.25 kilogram per cubic meter, *A* is the front area of the object (as seen from the direction it is heading; if it is falling, then this is the area looking up at it), *v* is the velocity (in meters per second). *F* is the force of air resistance in Newtons. (See footnote 2 if you want to convert to pounds of force.)

As the object falls faster and faster, the force of air resistance gets greater and greater. Gravity is pulling down, but the air is pushing up. The force of the air resists the gravity; it opposes it. If the object keeps on accelerating, eventually the force of air will match the weight of the object. When that happens, gravity and air resistance are balanced. The object doesn't stop moving, but it stops accelerating, that is, it no longer gains additional velocity. When this happens we say the falling object has reached *terminal velocity*.

For the food dropped over Afghanistan, the terminal velocity was about 9 mph. For a falling person it is typically 70-100 mph. That's fast, but people have survived falls from great heights into water. (Try to imagine this next time you are going in a car or train at 74 mph.) If a falling person spreads out his arms and legs (like sky divers) that increases the effective area, and the person will fall even slower. A person using a parachute is not much heavier, but the parachute area *A* is large, so the terminal velocity is only about 15 mph. For King Kong (about the same area as a parachute, but much heavier) the terminal velocity was several hundred mph; he hit the ground without ever reaching it.

It is interesting to think about why it goes at this speed. If the falling object went any faster, the upward force (from the air) would be greater than the weight, and that net force would slow the fall. If the packet fell any slower than this terminal velocity, then the downward pull of gravity would be stronger than the upward force of air resistance, and the packet would fall faster.

The terminal velocity equation.

If we take the air resistance equation and set $F = mg$(where m is the mass of the falling object) and solve for *v,* we get the equation for terminal velocity:

$$v = \sqrt{\frac{2mg}{A\rho}}$$

A food packet in Afghanistan had a mass of about m = 0.1 kg, and an area of about $A = 0.3m \times 0.3m = 0.1m^2$; g = 9.8 m/s^2. Putting in these numbers we get for the food packets:

$$v = \sqrt{\frac{2 \times 0.1kg \times 9.8m/s^2}{0.1m^2 \times 1.25kg/m^3}} \approx 4\tfrac{m}{s} = 9\,mph$$

That's about the speed at which you jog. So the food floats down relatively slowly, not at the high speed we had obtained when we neglected air resistance.

Let's calculate the terminal velocity for a person. We need to estimate the person's area and his mass. The only area that counts is the area hit by air as he is falling. If he is diving, that would be the area of the top of his head. If he is doing a swan dive, it would be the area of the front of his body, roughly his height times his width. To make the numbers simple, let's assume he is 2 meters tall and 1/2 meter wide, giving an area of $2m \times 0.5m = 1m^2$. Take his weight to be M = 160 lb = 70 kg. Then, according to our formula, his terminal velocity will be:

$$v = \sqrt{\frac{2mg}{A\rho}} = \sqrt{\frac{2 \times 70kg \times 9.81m/s^2}{1m^2 \times 1.25kg/m^3}} = 33\tfrac{m}{s} = 74mph$$

With a parachute, the mass of the falling person is about the same (parachutes are light and don't contribute much to the total weight), but the area subject to air resistance can be 30 times larger than the area of a person's body. Plug that into the equation and see how much it slows the falling person. (Hint: it will be $\sqrt{30} \approx 5.5$ times slower. Do you see why?)

What about King Kong? Did he slow down? The answer is no – and the reason is interesting. If he were 10 times taller than a person, his weight would be 1000 times more. (He is not only 10 times taller, but also 10 times wider and 10 times thicker; that makes his volume 1000 times larger.) But his area would have been only 100 times greater. (The area that the air it hitting, to slow him down, is width times height; it doesn't depend on his thickness.)

His weight is much bigger than that of a human, and his area (the feature that responds to air resistance) is not nearly that much bigger. Plug in those numbers (M = 70,000 kg and A = 100 square meters) and you'll see that King Kong would not have slowed down very much. That aspect of the movie was accurate.

Automobile Fuel Efficiency

A moving car feels the force of air on its front, and that tends to slow the car down. To keep going at the same velocity, the engine must make up the lost energy. We'll show that much of the gasoline used by the car is to overcome the force of this air resistance. That is an important fact to know. At a velocity of 30 m/s = 67 mph, the force of air on the front of the car will be about 500 lb!

To calculate air resistance on the front of a car, we apply the air resistance equation:

$$F = \tfrac{1}{2} A\rho v^2$$

The area A that we need to use in this equation is the area of the front of the car, equal to the height of the car times its width. Just to get a rough idea, assume the car is about 2 meters by 2 meters, giving A = 4 square meters. For the velocity let's take 67 mph = 30 m/s. The density of air is ρ = 1.25 kg per cubic meter. Plugging in these values, we get that the force on the front of the automobile is:

$$F = \tfrac{1}{2} \times 4m^2 \times 1.25kg/m^3 \times (30m/s)^2 = 2250N \approx 500lb$$

To keep the car from being slowed down by this force, the engine must exert an equal and opposite force. That takes a lot of gasoline. At these high velocities, more than 50% of the gasoline is used to overcome this air resistance. As a result, car designers have worked hard to "streamline" the shape of automobiles. If, instead of hitting the air with a flat surface (as in the old autos from the 1920s) you tilt the front surface, then the force can be much less, since the molecules can bounce off obliquely instead of hitting the front and bouncing straight back. In such a car, the force can be as low as 100 lb.

Many truck drivers are in business for themselves, and they have to pay for the extra gasoline used to overcome air resistance. Maybe you've noticed the smooth curves that some truck drivers have added to the cabs of their trucks to do the same thing. Reducing the force can save substantial money on gasoline.

Figure: Aerodynamic design for a truck cab
The top of the cab has had a contoured shape added to it to make the air bounce off smoothly, at an angle, instead of hitting the flat face of the truck head on. (The added shape is called "fairing.") This is shape change is sometimes called "aerodynamic smoothing" and it saves gasoline. It makes the effective value of "*A*", the area in the air resistance equation, smaller.

Note also that at half that speed (i.e. if v = 15 m/s = 33 mph) the force is four times less. That means that you use 4 times less gasoline to overcome air resistance. So you save even more gasoline by driving slower.

Force and Energy

The relationship between energy and force is remarkably simple: *energy is force times distance*. The equation is:

$$E = F \times D$$

This means that if you push something with a force F (in Newton) for a distance D (in meters), then the energy it takes is E, joules. How much energy does it take to go up a flight of stairs? Suppose you go up 4 meters (about 13 feet) and you weigh 160 lb (or 714 N). Then the work you do is

$$E = 714N \times 4m \approx 3000J = 0.7Cal$$

That's why it's hard to lose weight through exercise. Drink one 12 ounce can of cola, which typically has 140 calories, and you can work it off by going up 200 flights of stairs! Food contains a great deal of energy. That's why you can get all the calories you need per day from about 2 lb of food. (1 kg of whole wheat bread contains about 2500 Cal.)

Force and Acceleration

If you push on an object, and there is no friction to hold it back, then it gains velocity. If you push twice as hard, it gains twice as much velocity. The acceleration is proportional to the force. But it also depends on the mass of the object. If the object has twice the mass, it needs twice the force to get it moving. These two facts are summarized the following equation:

Newton's Second Law:

$$F = ma$$

In this equation, m is the mass (in kg), a is the acceleration (in m/s^2) . The force is then in *Newtons*[13].

This equation tells you how much things speed up – or slow down -- when you apply a force.

This is Newton's Second Law. You may be wondering what Newton's *First* Law is. It states that unless an object has an outside force on it, it will tend to keep its motion unchanged. But that is just a special case of the Second Law, since if $F = 0$, there is no acceleration, and that means no change in velocity. Nobody would ever teach the First Law these days if not for the fact that students sometimes wonder what came before the Second Law.

The g

In this book, accelerations are typically measured in m/s per second (m/s^2). For automobiles, we frequently measure acceleration in miles per hour every second, sometimes written as mph/sec. For example, if a car salesman tells you that a car will "go from zero to sixty in ten seconds" what he is really telling you is the acceleration: 60 mph in 10 seconds. That is the same as 6 mph every second = 6 mph/s.

Another very useful unit for acceleration, used in the military and by NASA, is the "*g*", pronounced "gee." One *g* is 9.8 meters/sec every second, i.e. it is the acceleration of gravity. Suppose an automobile accelerated at one *g*. How fast would it be going in 10 seconds? The answer is found from our velocity equation by setting $a = 9.8\,\text{m/s}^2$:

[13] Optional footnote: Some physics texts will take this equation as the definition of mass. In that case, Newton's Second Law can be stated as follows: "the mass of an accelerating object is approximately constant." As we will see in the chapter on Relativity, at extremely high velocities, this law breaks down; mass increases with velocity.

$$\begin{aligned} v &= at \\ &= 9.8\,\tfrac{m}{s^2} \times 10s \\ &= 98\,\tfrac{m}{s} \\ &= 219\,\text{mph} \end{aligned}$$

According to a NASA web site, when the Space Shuttle is launched, the acceleration reaches a maximum of 3 *g*. This is usually pronounced as "three gees." You've probably heard this term used in news programs and in movies. A pilot in an F-18 fighter plane sometimes accelerates as much as 10 *g*, i.e. "ten gees." Ten *g* would be $10 \times 9.8\,\text{m/s}^2 = 98$ meters/sec every second. Weightlessness is often described as "zero *g*." That's when there is no acceleration – at least with respect to your space capsule.

When something accelerates at $a = 2g$, we'll sometimes say it in text as "the acceleration is two *g*s." That can be said aloud as "the acceleration is two gees."

Why is the acceleration of gravity a constant?

Why does gravity give a constant acceleration g? It turns out that this law is a consequence of combining Newton's Law of Gravity and Newton's Second Law. The force of gravity near the surface of the Earth is given by the equation

$$F = G\frac{mM}{r^2}$$

In this equation, $r = R_E$ is the distance to the center of the Earth, and $M = M_E$ is the mass of the Earth. According to Newton's Second Law, this force will causes an acceleration given by $F = ma$. Let's set these two equations for *F* equal to each other:

$$F = m\,a = G\frac{m\,M_E}{{R_E}^2}$$

If we solve for *a* we get:

$$a = G\frac{M_E}{{R_E}^2}$$

But the right hand side of the equation has nothing but constants: *G* is the gravitational constant, M_E is the mass of the Earth, and R_E *i*s the radius of the Earth. If we put in the appropriate numbers we get:

$$a = 9.8\,\tfrac{m}{s^2}$$

That is just the value that we previously called *g*. It is the "acceleration of gravity."

So here is an "explanation" of why all objects fall with the same acceleration. More massive objects need a bigger force to get accelerated; that is Newton's second Law. But they have a bigger force – their weight! So big objects will accelerate at the same rate as less massive objects.

The g-rule

There is a very good reason to think of acceleration in terms of *g*s: it enables you to solve important physics problems in your head. Suppose you are accelerated in the horizontal direction by 10 *g*s. How much force will that take? The answer is simple: 10 times your weight! When astronauts are accelerated by 3 *g*s, the force on them to do this must be 3 times their weight. We can call this the "*g*-rule": *the force to accelerate an object is equal to the number of gs times the weight*

To get the number of *g*s, just calculate the acceleration in m/s per second, and divide by 9.8. So, for example, we write that an acceleration of a = 19.6 m/s per second = 2 g. This is an acceleration of two *g*s. To accelerate something to 2 *g*s takes a force equal to twice the weight of that something.

Why the g-rule is true

The force required to accelerate an object of mass m is given by Newton's Second Law:

$$F = m\,a$$

The number of gs is

$$N = a/g$$

so

$$N\,g = a$$

The weight of the object is

$$w = m\,g$$

We now use all of these equations:

$$\begin{aligned} F &= m\,a \\ &= m\,g\,N \\ &= w\,N \end{aligned}$$

The last equation is the g-rule.

The distance equation

In the beginning of this chapter, we said that the distance you fall in a time t, under the influence of gravity, is

$$D = \tfrac{1}{2} g t^2$$

The same formula will apply when you undergo any constant acceleration a, if you simply replace the symbol g with the symbol a. So the distance you travel will be:

$$D = \tfrac{1}{2} a t^2$$

We can write this in terms of the final velocity v by substituting $t = v/a$:

$$\begin{aligned} D &= \tfrac{1}{2} a t^2 \\ &= \tfrac{1}{2} a \left(\frac{v}{a}\right)^2 \\ &= \tfrac{1}{2} \frac{v^2}{a} \end{aligned}$$

If we solve this equation for a, we get

$$a = \frac{v^2}{2D}$$

This equation tells you the acceleration you need to reach a velocity a in a distance D.

Shooting an astronaut into space: the rail gun

To go into orbit around the Earth, a satellite must have a velocity of 8 km/sec. Why not give it this velocity in a gun? Could we literally "shoot" an astronaut into space?

The answer is: you might be able to do this, but the astronaut would be killed by the force required to accelerate him. Let's assume we have a very long gun, an entire kilometer long. If we use the equation in the last section, we derive that the required acceleration is $a = 3200\ g$, i.e. the acceleration is 3200 times the acceleration of gravity.

Calculation: 1 km gun shooting to 8 km/sec.
We can calculate this by using the last formula in the last section. The hardest part is getting the units right. The distance $D = 1$ km = 1000 meters, and the velocity $v = 8$ km/sec = 8000 m/sec. Plugging these in, we get:

$$\begin{aligned} a &= \frac{v^2}{2D} \\ &= \frac{(8000\frac{m}{s})^2}{2 \times 1000 m} \\ &= 32000\frac{m}{s^2} \\ &= \frac{32000\frac{m}{s^2}}{9.8\frac{m}{s^2}} \times g \\ &\approx 3200 g \end{aligned}$$

That's a lot of *g*s. Remember the *g*-rule: the force that it will take to accelerate you this fast will be 3200 times your weight. So if you weigh 150 lb, the force that must be applied to you is 3200×150 lb = 480,000 lb = 240 tons. That is enough to crush your bones.

Suppose we want to experience no more than 3 *g*s – that's what the Space Shuttle does. How long would the gun have to be to get you going at v = 8 km/s? The gun would have to be 1000 km long! That is, of course, ridiculously impractical.

Calculation: We first must convert 3 *g*s to metric units: $3g = 3 \times 9.8 \frac{m}{s^2} = 29.4 \frac{m}{s^2}$.
Now put this into the distance equation:

$$\begin{aligned} D &= \frac{v^2}{2a} \\ &= \frac{(8000 \frac{m}{s})^2}{2 \times 29.4 \frac{m}{s^2}} \\ &= 1.1 \times 10^6 m \\ &= 1100 km \end{aligned}$$

Of course, acceleration over such long distances is not impractical – it's what the Space Shuttle does. It takes off, accelerates at 3 *g*s, and it must go over 1000 km to reach orbital speed. So it is like a very long gun, but without a barrel. It actually takes the Space Shuttle further than 1000 km since 3 *g*s is only the peak acceleration; for most of the flight, the acceleration is less.

Acceleration during airplane take-off

The take-off speed for a commercial airplane is about 160 mph. What acceleration is needed to achieve this speed on a 1 km runway? Let me make that question a little more personal. You are sitting in such an airplane. In a few seconds, you will be at the other end of the runway, and you will be moving (along with the airplane) at 160 mph. What force does the seat have to apply to your back to accelerate you?

We show below that this requires the airplane to accelerate at 2.5 m/s per second. We can convert this number to *g* units by dividing by 9.8, to get that the acceleration is 2.5/9.8 g = 0.26 *g*. So the force that the airplane must push on you to get you going that fast is 0.26 times your weight, i.e. about a quarter of your weight. If you weigh 150 lb, then the push you feel on your back will be about 39 lbs. Think of this next time you are on an airplane taking off. Does this number feel about right?

Calculation: To calculate the acceleration of the airplane, we first convert to metric units. We divide the lift-off speed of 160 mph by 2.24 to get $v = 71$ m/s. We take $D = 1$ km = 1000 meters and put it into the acceleration equation:

$$\begin{aligned} a &= \frac{v^2}{2D} \\ &= \frac{(71\,\frac{m}{s})^2}{2\times 1000m} \\ &= 2.5\,\frac{m}{s^2} \\ &= \frac{2.5}{9.8}g \\ &= 0.26g \end{aligned}$$

Circular acceleration

Physicists like to define velocity as having magnitude and direction. If your velocity changes, we call it acceleration. But suppose you only change your direction and not the actual number of m/s? We still call that acceleration.[14] The reason we do that is that many of the equations we've been using still work.

The most important example of this kind of acceleration is when you go in a circle. If the magnitude of your velocity is v, and this doesn't change (you keep going the same number of m/s or mph), and the circle has radius R, then we say that the circular acceleration is

$$a = \frac{v^2}{R}$$

This kind of acceleration is very important for the fighter pilot who is trying to change his direction rapidly. For example, if he is moving at velocity of 1000 mph, and is turning in a circle of radius R = 2 km, then his acceleration turns out to be 10 g. That's about as much as a fighter pilot can tolerate.

[14] For people who have studied vectors: velocity is defined in physics as a vector. If the velocity at time t_1 is $\mathbf{v_1}$, and at t_2 is $\mathbf{v_2}$, then the acceleration vector is defined as $\mathbf{a} = (\mathbf{v_2}-\mathbf{v_1})/(t_2-t_1)$. Even if the velocity is only changing in direction, the difference vector $(\mathbf{v_2}-\mathbf{v_1})$ is not zero. For circular motion, its magnitude is given by the equation in the text.

Calculation: Acceleration of fighter pilot.
First we convert these to metric units. We convert mph to m/s by dividing by 2.24, giving v = 446 m/s. We also convert R to meters: R = 2000 meters. Plugging these in gives:

$$
\begin{aligned}
a &= \frac{v^2}{R} \\
&= \frac{(446\tfrac{m}{s})^2}{2000m} \\
&\approx 100\tfrac{m}{s^2} \\
&= \tfrac{100}{9.8}g \\
&\approx 10g
\end{aligned}
$$

The problem with accelerating more than 10 g is that a human's blood pressure is not great enough to keep the blood in the brain, and that causes the pilot to faint. Even specially chosen and physically fit fighter pilots cannot take more than 10 g.

Fighter pilots and astronauts are usually tested and trained in a spinning cylinder, rotated fast enough for them to experience 10 g circular acceleration.

High g, from circular acceleration, is also a method used to separate the components of uranium for making a nuclear weapon. Such a device is called a centrifuge. The heavier parts of the uranium feel a greater force, and they are pulled more strongly towards the outer parts of the spinning cylinder. Such centrifuges are much in the news. In 2004, Libya disclosed the fact that it had purchased large centrifuge plants for the production of nuclear weapons material. Parts of such centrifuges have been found hidden in Iraq. We'll talk more about these centrifuges when we get to the chapter on nuclear weapons.

Gravity in Space – according to science fiction

Science fiction movies sometimes show a large rotating space station consisting of one or more large tubular rings (or donuts) connected to one another. Astronauts are shown living and working in the tubular rings as they would on Earth using the "artificial gravity" from the rotation. This actually makes sense. The astronauts will feel a force on their feet (which point towards the outside) that will appear to them indistinguishable from gravity. Such a satellite is shown in the classic movie *2001 – A Space Odyssey*. We'll show below (see the calculation) that the edge of a 200 meter-radius satellite must be moving at about 44 meters/sec. That means it will rotate once every 40 seconds.

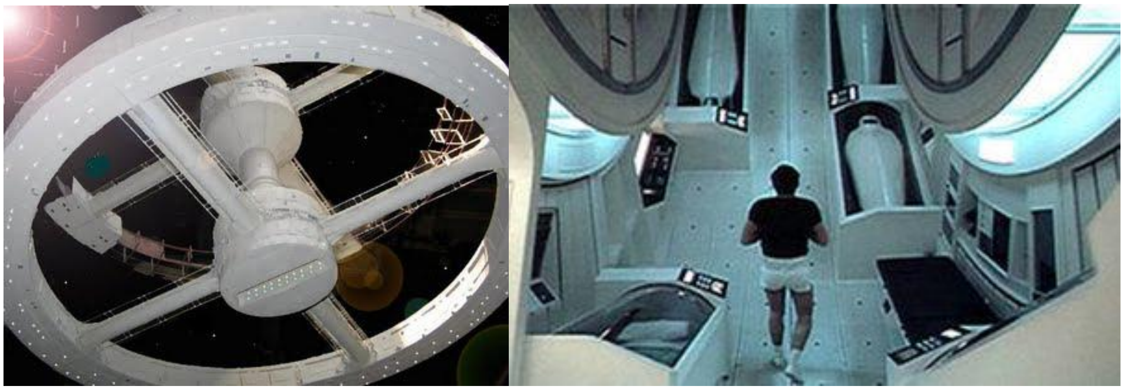

Figure: Scenes of space station (left) and jogger running with artificial gravity (right) from *2001: Space Odyssey.*

Calculation:
If the satellite has radius R and is rotating with a rim velocity v, then the force on the astronauts feet will be

$$F = ma = m\frac{v^2}{R}$$

To make this equal to the astronaut's weight on Earth, we set $F = mg$. Solving for v, we get

$$v = \sqrt{gR}$$

For a satellite with a radius of 200 meters, the rim velocity must be

$$v = \sqrt{9.8\tfrac{m}{s^2} \times 200m} = 31\tfrac{m}{s}$$

Artificial gravity in space – without circular motion

Many science fiction movies show space voyagers in the space ships walking around as if on the Earth. Is that nonsense? Where does that gravity come from?

We can make some sense out of it. Just assume that the space ships are not moving at constant speed. If the ship engine accelerates the ship by a = 9.8 meters per second, then the ship will put a force on any astronauts inside to accelerate them too. A person of mass m will feel a force $F = ma$. But since the engines have set $a \approx g$, the force on the astronaut will be $F = m\,g$. That is exactly his Earth weight. If he places his feet on the backside of the ship (opposite in direction to the path the ship is taking) then he will feel like he is standing on Earth. In fact, he can't even tell the difference; the acceleration serves as a "virtual" gravity.[15]

Here is an interesting number: the distance the ship would travel in a year if it had constant acceleration of g. See the calculation below. The answer is 5 x10^{15} meters, which is about half a light year (the distance that light travels in a year). The distance to the nearest star (not counting the sun) is about 4 light years.

Calculation: The distance that you travel at acceleration *g* is given by our distance equation: $D = \frac{1}{2} g t^2$. We put in $g = 9.8 \frac{m}{s^2}$. For mks units, we need to have *t* in seconds. We can calculate the number of seconds in a year as follows: 1 year = 365 days = 365 x 24 hours = 365 x 24 x 60 minutes = 365 x 24 x 60 x 60 seconds = 3.16 x10^7 seconds. Plugging in this value we finally get $D = \frac{1}{2} 9.8 \frac{m}{s^2} (3.16 \times 10^7)^2 = 5 \times 10^{15}\, m$. One light year is the speed of light (3 x10^8 m/s) times the number of seconds in a year (which we just showed was 3.16 x10^7 seconds). That gives 9.5 x10^{15} meters.

Escape to Space

Suppose you want to completely leave the Earth. Just going into orbit isn't enough; you want to get away completely, maybe to take a trip to the Moon, Mars, or a distant star. That takes more energy than just going into orbit. How much more? The answer is surprisingly simple: exactly twice as much. You can get that much kinetic energy by going 1.414 times faster than orbital speed. (That's because energy depends on the square of the velocity, and $1.414^2 = 2$.) Since orbital velocity is 8 km/sec, that means that escape velocity is $1.414 \times 8 \frac{km}{s} \approx 11 \frac{km}{s}$.

Equations for escape energy and escape velocity.
If you know some calculus, then you can understand that the energy is $E = \int F\, dr$. If you put in $F = G \frac{mM}{r^2}$ and integrate from $r = R_E$ (the Earth's radius) to $r = \infty$ (infinity), then you get the result $E = \frac{GmM}{R_E}$. This is the equation for the escape energy. For the escape velocity, set this energy equal to $E = \frac{1}{2} m v^2$ and solve for *v*. This gives the escape velocity:

$$v = \sqrt{\frac{2GM}{R_E}}$$

If we put in the values for *G, M* (the mass of the Earth) and R_E, then we get the value v = 11000 m/s = 11 km/s.

[15] Movies sometimes show the virtual gravity as being off to the side. This could be accomplished by having special sideways thrusters to accomplish this. Every hour or so the thrusters could rotate the ship around so it doesn't wind up going too far sideways.

At the escape velocity, the kinetic energy of a 1 kg object is equal to 15 Calories per gram.[16] That is only a little bigger than the energy of gasoline – but as soon as you are a few kilometers above the surface of the Earth, you begin to run out of oxygen, and gasoline needs oxygen. If you carry the oxygen with you (and rockets do that), it greatly increases the weight. That's why it is so hard to get objects into space. If you use chemical fuel, then the fuel weighs substantially more than the object you are sending up. Typically, you have to use 20 grams of fuel to launch 1 gram of satellite; equivalently, you have to use 20 tons of fuel to launch 1 ton of satellite. That's why rockets are so much larger than the satellites they launch.

It is very interesting that the escape velocity does not depend on the mass. More massive objects do require more energy to escape, but at the same velocity, more massive objects already have more energy! Remember $E = \frac{1}{2}mv^2$. At escape velocity v = 11 km/sec, and object with 2 kg mass has twice the kinetic energy as an object of 1 kg.

Black Holes[17]

Big planets have high escape velocities. For Jupiter, it is 61 km/sec. For the sun it is 617 km/sec. Are there any objects for which the escape velocity is higher than the speed of light, $3\times10^5 \frac{\text{km}}{\text{s}} = 3\times10^8 \frac{\text{m}}{\text{s}}$? The surprising answer is *yes*. We call such objects *black holes*. They get their name from the fact that even light cannot escape, so we can never see their surface; they are black. We'll show in Chapter 11 that no ordinary object (made out of mass as we know it) can go faster than the speed of light. We'll see in Chapter 11 that nothing can go faster than light. That means that nothing could escape a black hole.

If it's invisible, how do we know it's there? The answer is that even though it can't be seen, we can see the effects of its very strong gravity. When there is nothing visible, but the gravity is so strong that we know there must be something of great mass present, then we deduce it must be a black hole.

To be a black hole, you have to do one of two things: either have a lot of mass, or pack a moderate amount of mass into a very small radius. Several black holes are known to

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[16] $E = \frac{1}{2}mv^2 = \frac{1}{2}\times 1kg \times (11000\frac{m}{s})^2 = 6\times10^7 J$. This is 15000 Calories per kg, equal to 15 Cal per gram.

[17] Why is the physics of black holes included in this text? Is there any practical use for this knowledge? The answer is: no, not really. It is included, as are a few other things in this book, just because most people have heard about them and are curious. But you never know. Knowing the size of a black hole once won the author of this book a free guide to Paris. Outside of Shakespeare Books on the west bank of the Seine was a sign offering this prize if anyone could answer the question, "What size would the Earth have to be for it to be a black hole?"

exist; even though we can't see them (no light leaves the surface) we know that they are there because of the strong gravitational force they exert. The known black holes are all as massive as a star – or greater. But if you were to take the mass of the Earth and pack it inside a golf ball (in principle this is possible), then it would be a black hole. The mass would be the same, but the radius would be so small that the gravitational force on the surface of the golf ball would be enormous.

The sun is more massive, so you don't have to pack it so tightly. It would be a black hole if you packed it inside a sphere with radius 2 miles.

These are calculated by using the "black hole equation." This equation comes from taking the equation for the escape velocity and setting it equal to the speed of light, $c = 3\times 10^8 \, \frac{\text{m}}{\text{s}}$.

The black hole equation

If R is the radius of the black hole (usually called the "Schwarzschild radius") and M is its mass, then we get the black hole equation by taking the escape velocity equation and setting $v = c$, the speed of light. (Strictly speaking, this derivation is not correct since we are assuming that the mass is independent of velocity. We are also assuming ordinary "Euclidean" geometry). When we do this we get:

$$c = \sqrt{\frac{2GM}{R}}$$

Solving for R, this gives the Black Hole Equation:

$$R = \frac{2GM}{c^2}$$

Any object of mass M that has all that mass in a sphere of radius R or smaller, will be a black hole. If we take $M = M_E$ (the mass of the Earth), then we conclude that the Earth would be a black hole if its radius were less than

$$\begin{aligned} R &= \frac{2GM_E}{c^2} \\ &= \frac{2\times 6.67\times 10^{-11}\,\frac{Nm^2}{kg^2}\times 6\times 10^{24}\,kg}{(3\times 10^8\,\frac{m}{s})^2} \\ &= 9\times 10^{-3}\,m \\ &= 0.9\,cm \end{aligned}$$

Thus, the Earth would be a black hole if all its mass were stuffed into a golf ball. For the Sun to turn into a black hole, its mass ($M_S = 2 \times 10^{30}$ kg) would have to be stuffed in a sphere of radius

$$\begin{aligned} R &= \frac{2GM_S}{c^2} \\ &= \frac{2 \times 6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \times 2 \times 10^{30} kg}{(3 \times 10^8 \frac{m}{s})^2} \\ &= 3000m \\ &\approx 2\,\text{miles} \end{aligned}$$

So, if the Sun were squeezed into a radius of 2 miles it would be a black hole.

There are several black holes that were created, we believe, when the inner part of a star collapsed from its own weight into a very small radius. The object in the sky known as Cygnus X-1 is thought to be a black hole from such a collapse. If you have spare time, look up Cygnus X-1 on the web and see what you find.

Many people now believe that large black holes exists at the center of the large collections of billions of stars known as galaxies, such as the Milky Way galaxy. We presume these were created when the galaxy formed, but we know almost none of the details about how this happened.

Even more remarkably, the Universe itself may be a black hole. That's because the black hole radius for the Universe is about 15 billion light years, and that is approximately the size of the observable Universe. In other words, the Universe appears to satisfy the black hole equation. We'll discuss this further in Chapter 11. But you probably won't be surprised at one inescapable conclusion that follows[18]: we can never escape from the Universe.

Momentum – and Newton's Third Law

If you shoot a powerful rifle, then the rifle puts a large force on the bullet sending it forward. But the bullet puts a backward force on the rifle, and that is what causes the "kick." The rifle can suddenly go backwards so rapidly that it can hurt your shoulder. If you don't have your feet firmly planted on the ground, you will be thrown backwards.

This effect is sometimes given the fancy name "Newton's Third Law", and it is stated as "for every action there is an equal and opposite reaction." But we no longer use this old terminology of "action" and "reaction." Instead we say that if you push on an object

[18] pun intended.

(such as a bullet), then the bullet also pushes back on you for exactly the same amount of time. Of course, the bullet is lighter, so it is accelerated much more than you are.

Based on the fact that the bullet pushes on the rifle for the same time that the rifle pushes on the bullet, we can derive[19] an extremely important equation, sometimes called the *conservation of momentum*. For the rifle (subscripts R) and the bullet (subscripts B), the equations are:

$$m_R v_R = m_B v_B$$

The equation for the rifle recoil is simple: the mass of the bullet times its velocity, is the same as the mass of the gun times its recoil velocity. Of course, the velocities are opposite; this is sometimes indicated by putting a minus sign in front of one of the velocities. (We didn't do that here.) When the gun is stopped by your shoulder, then you recoil too – but less, because you have more mass.

The product mv is called the "momentum." One of the most useful laws of physics is called "the conservation of momentum." Before the rifle was fired, the bullet and gun were at rest; they had no momentum. After the rifle fired, the bullet and the rifle were moving in opposite directions, with exactly opposite momenta (the plural of momentum), so the total momentum was still zero.

Here's another way to say that: when you fire a gun, the bullet gets momentum. You and the rifle you are holding get an equal and opposite momentum. If you are braced on the ground, then it is the Earth that recoils with that momentum. Because the Earth has large mass, its recoil velocity is tiny, and difficult to measure.

If the objects are in motion prior to the force acting, then what happens is that the *changes* in momentum must be equal and opposite. Let's apply that to the comet that crashed into the Earth and killed the dinosaurs. To make the calculations easy, assume that before the collision the comet with mass m_C was moving at v_C = 30 km per second (a typical velocity for objects moving around the Sun). Assume the Earth was at rest. After the collision, the total momentum would be the same. The Earth would have mass m_E (that now included the mass of the comet) and have velocity v_E. So we can write

$$m_C v_C = m_E v_E$$

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[19] The derivation is based on calculus. If an object is at rest, and it experiences a force F for a time t, then its velocity $v = \int a\,dt = \int \frac{F}{m}\,dt$. Write this as $mv = \int F\,dt$. If there are two objects, and the forces are equal and opposite, and the time is exactly the same, then $\int F\,dt$ is equal and opposite for the two objects, and that means that the quantity mv must also be equal and opposite for the two objects.

Solving for v_E we get

$$v_E = \frac{m_C v_C}{m_E}.$$

The mass of the comet is about 10^{19} kg.[20] Everything else is known, so we can plug into this equation and get v_E, the velocity of recoil of the Earth:

$$\begin{aligned} v_E &= \frac{m_C v_C}{m_E} \\ &= \frac{10^{19} kg \times 3\times 10^4 \frac{m}{s}}{6\times 10^{24} kg} \\ &= 0.5 \frac{m}{s} \end{aligned}$$

That's not much of a recoil, at least when compared to the usual velocity of the Earth, which is 30 km/sec = 30,000 meters/sec. So the Earth was hardly deflected. It's orbit changed, but only by a tiny amount.

Let me estimate how much a truck recoils when it hits by a mosquito. Assume (from the point of view of the truck, the mosquito (weighing 2.6 milligrams) is moving at 60 mph = 27 meters per second. Assume the truck weighs 5 metric tons = 5000 kg. (Hint: use the same equation, except let the 2.6 milligram mosquito represent the comet. Don't forget to convert everything to the same units: kilograms and meters.)

Although the conservation of momentum is one of the most important laws of physics, it is violated in many action movies. For example, if the hero in the Matrix punches the villain, and the villain goes flying across the room, then the hero should go flying backwards (unless he is braced on something big and massive). Likewise, small bullets, when they hit a person, seem able to impart very large velocities to the person that they hit, so the person goes flying backwards.

Rockets

Imagine trying to get into space by pointing a gun downward and firing bullets so rapidly that the recoil pushes you upward. Sounds ridiculous? Yet that is exactly how rockets work.

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[20] A comet with a radius $R_C = 100\text{km} = 10^5$ m, has a volume of $V_C = \frac{4}{3}\pi R_C{}^3$ (for a sphere) = 4.2×10^{15} cubic meters. Assuming that the comet is made mostly of rock and ice, the density is probably about 2500 kg per cubic meter, so the mass is $2500 \frac{kg}{m^3} \times 4.2\times 10^{15} m^3 = 10^{19} kg$.

Rockets fly by pushing burned fuel downward. If the fuel has mass m_F and is pushed down with a velocity v_F, then the rocket (which has mass m_R) will gain an extra upward velocity v_R given by the same kind of equation we used for the rifle:

$$v_R = \frac{m_F v_F}{m_R} = \frac{m_F}{m_R} v_F$$

Compare this to the rifle equation, and to the comet/Earth collision equation.

Because the rocket weighs so much more than the fuel which is expelled every second (i.e. m_F / m_R is tiny), the amount of velocity gained by the rocket is much less than the fuel velocity. As a consequence, rockets are a very inefficient way to gain velocity. We use them to go into space only because in space there is nothing to push against except expelled fuel. Another way to think of this is as follows: rockets are inefficient because much of the energy goes into the kinetic energy and heat of the expelled fuel, rather than in the kinetic energy of the rocket.

The equation above gives the velocity *change* when a small amount of fuel is burned and expelled. To get the total velocity given the rocket, you have to add up a large number of such expulsions. Meanwhile, the mass of the rocket (which is carrying the unused fuel) is changing as fuel is used up. A typical result is that the rocket must carry huge amounts of fuel. The mass of fuel used is typically 25 to 50 times larger than the payload put into orbit, as shown below.

Calculation: rocket fuel

If we assume that the rocket started at rest, then the final velocity of the rocket is given by the rocket equation, which is derived in the footnote[21]

$$v = 2.3\, v_F \log\left(\frac{m_R}{m_F}\right)$$

This can also be written in terms of the amount of fuel needed:

$$m_F = m_R \times 10^{v/(2.3 v_F)}$$

In this equation m_F is the initial mass of the rocket and fuel (mostly fuel) and m_R is the final mass (mostly rocket).

[21] If you know some calculus, then you can calculate the final velocity as follows. In the notation of calculus, the change in velocity (which we called v_R in the text) is written as dv, and the mass of fuel consumed (which we called m_F in the text) is dm, and the mass of the rocket is m. The final velocity $v = \int dv = \int v_F \frac{dm}{m} = v_F \ln\left(\frac{m_R}{m_F}\right) \approx 2.3 v_F \log\left(\frac{m_R}{m_F}\right)$.

QED.

Assume[22] now that the burned fuel expulsion velocity is 3 km/sec, and that the orbital velocity that we want to achieve is 8 km/sec. Then, according to this equation, the mass of fuel needed to reach orbital velocity is

$$\begin{aligned} m_F &= m_R \times 10^{8/(2.3\times 2.5)} \\ &= m_R \times 10^{1.4} \\ &= 24\, m_R \end{aligned}$$

Thus, for this example, the fuel carried by the rocket must weigh 24 times as much as the payload!

For a long time, this huge fuel to payload ratio led people to believe that rockets into space were impossible; after all, how could you even hold the fuel if it weighed 24 times as much as the rocket? The problem was solved by using rockets with multiple stages, so that the heavy containers that held the initial fuel never had to be accelerated to the final orbital velocity. For example, for the space shuttle, the final payload (including orbiter weight) is 68,000 kg = 68 tons, and the boosters plus fuel weighs[23] 1931 tons, a factor of 28 times larger. Of course, the boosters never get into orbit, only the much smaller shuttle.

Balloon Rockets and Astronaut Sneezes

When you inflate a balloon, and then release the end, the balloon goes whizzing around the room, driven by the air being pushed out the end. What is happening is nearly identical to the way a rocket flies. Before you release the balloon opening, the balloon and the air have zero total momentum. When you let the air come out, it rushes out with high speed. It is pushed by the compressed air in the balloon, and it pushes back. It pushes back on that air, and that air pushes on the balloon. The released air goes one way, and the balloon containing its remaining air goes the other way, and the two momenta cancel.

[22] Optional calculation for those interested: If the fuel carries 1 Cal per gram, and all that energy is converted into kinetic energy, then we can calculate the velocity by assuming it all goes to kinetic energy. 1 Cal = 4200 joules. So 1 kg of fuel would have 4,200,000 joules. If it is all converted to kinetic energy, we would have $\frac{1}{2}mv^2 = 4{,}200{,}000\,J$. Using m = 1 kg, this gives $v^2 = 8{,}400{,}000\,\frac{m^2}{s^2}$. Take the square root to get $v = 3000\,\frac{m}{s} = 3\,\frac{km}{s}$. Because chemical energy is not efficiently converted to kinetic energy (much remains as heat) the exhaust velocity of 2 km/sec is more typical of what is achieved.

[23] The external tank hold 751 tons of fuel, and there are two solid rocket boosters that weigh 590 tons each, for a total of 1931 tons.

When you sneeze, the sudden rush of air outward likewise can push your head backwards. In the opening of this chapter, there was a puzzle: how much does the astronauts head snap back. There was once a newspaper article claiming that the head would snap back at a dangerous speed because the head was weightless in space. That's not true, of course. The writer of the article had confused weight with mass. The astronaut's head has no weight but it has every bit as much mass as it did on Earth. The force of the sneeze will accelerate the head backwards by an amount given by $F = ma$, but since m is the same as on Earth, the acceleration a will be no greater.

Skyhook

Ponder the Space Shuttle. To put 1 kg payload into space requires 28 kg (fuel + container + rocket). Suppose, instead, we built a tower[24] that went all the way up to space. How much energy would it take to haul the ton up to the top, using an elevator? According to the section "Escape to Space", the energy required to take a gram of material to infinity is 15 Calories. That's roughly the energy in 1.5 grams of gasoline (not including the oxygen). So if we had a tower with the elevator, ran the motor below (where it could use air) getting to space would take 28/1.5 = 19 times less fuel.

Many people have pondered the fuel waste from rockets. Although a tower to space seems impossible, it may make sense to hang a cable down from a geosynchronous satellite and use it to haul payload up, an idea once referred to as "project skyhook." The recent discovery of very strong carbon nanotubes has revived the idea. Arthur C. Clarke used this idea in his 1977 science fiction novel "Fountains of Paradise."

A more likely idea is to "fly" to space on an airplane. Airplanes have two attractive features: they use oxygen from the atmosphere as part of their fuel (so they don't have to carry it all, as do rockets) and they can push against the air, instead of having to push against their own exhaust. Although it is possible in principle, the technology to achieve 8 to 12 km/sec with airplanes does not yet exist. We'll talk more about airplanes in a moment.

Earlier in this chapter we mentioned the possibility of using "rail guns." These are long devices that achieve their high projectile velocities by using electric and magnetic forces to push on the projectile. But recall their limitation: they must be very long in order to avoid huge accelerations that could kill humans.

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[24] In the Bible, such a tower was attempted in ancient Babylon, and it is also referred to as the "Tower of Babylon." Its goal was to reach heaven. To prevent the Babylonians from succeeding, God made all the workers speak different languages. Thus, according to the Bible, is the origin of the multitude of languages spoken by humans. It is also the origin of the verb "to babble."

Ion Rockets

The inefficiency of rockets comes from the fact that typical chemical fuels only have enough energy to give their atoms a velocity of 2-3 km per second. Rockets have been proposed that overcome this limitation by shooting out *ions*, a name for atoms that have an electric charge. You can find out a lot about these on the web. Like the rail gun, the ions can be given their high velocity through electric forces, so they are not limited to the 2-3 km/sec typical of rocket fuel. For example, a proton expelled at an energy of 100,000 eV ($1.6 \times 10^{-14} J$) has a velocity of 4400 km/sec. This makes ion rockets potentially much more efficient than chemical rockets, but so far nobody has figured out how to make the mass of the expelled ions sufficiently great to be able to launch a rocket from the Earth. They might be used for long duration trips between planets, or (if we ever do that) between stars.

Flying: airplanes, helicopters, and fans

Airplanes fly by pushing air downwards.[25] Every second, the airplane tends to pick up downward velocity from the Earth's gravity. It stays at the same altitude by pushing enough air downward that it overcomes this velocity.

The fact that wings push air down is most readily observed in a "rotary wing" aircraft, otherwise known as a helicopter. (Helicopter pilots called the ordinary airplane a "fixed wing" aircraft.) In fact, the helicopter blades are designed to have a shape identical to wings, and air is pushed past them when they spin. They push the air down, and that is what pushes the helicopter upward. If you stand under a helicopter rotor when it is spinning, you can feel the air being forced downward.

It is probably more convenient, however, to observe how wing-shaped blades push air by standing in front of a fan. Fans work the same way as helicopter blades, and as airplane wings. Movement through the air forces air perpendicular to the direction of motion of the blade.

For the airplane and the rocket, the v_R needed is the velocity to overcome the pull of gravity. In once second, gravity will give any object a velocity of

$$v = g\, t = 10 \text{ m/s}$$

[25] In most physics books, the lift on the airplane wing is explained by use of a principle called Bernoulli's law. The "derivation" is done using a diagram that typically shows the air trailing the wing as if it is completely undisturbed. The astute student will be bothered by this. How can the air put a force on the wing but the wing not put a force on the air? A careful analysis (done in advanced aerodynamics books) shows that to establish a flow with higher velocity above the wing than below, the far field air velocity distribution is not undisturbed; it fact, it has air deflected downward with a momentum rate equal to the upward force on the wings, as it must to satisfy momentum conservation.

This falling velocity must be cancelled by accelerating upward, and this is done in an airplane by pushing air downwards. Air is typically a thousand times less dense than the airplane (1.25 kg/m^3), so to get enough air (i.e. to make m_{air} large) the wings must deflect a large amount of air downward.

The wake of a large airplane consists of this downward flowing air, often in turbulent motion. It can be very dangerous for a second plane if it encounters this wake, since the amount of air flowing downward is large.

Flying: balloons

The first way that humans "flew" was in hot air balloons, in 1783 above Paris. These make use of the fact that hot air expands. It takes more volume compared to an equal mass of cool air. Another way to say this is that the density of hot air (the mass per volume) is less than that of cool air.

In a liquid or gas, things that are less dense tend to float. That's why wood floats on water (if it has a density less than one gram per cubic centimeter; some woods sink). The heavier fluid "falls" and flows under the less dense object, pushing it upward. Boats float only if their average density (metal hull plus empty space inside) is less than that of water. That's why a boat will sink if it fills with water.

If you fill a balloon with hot air, it will rise until it reaches an altitude at which the density of the surrounding air matches that of the balloon. (Of course, you have to include the mass of the balloon, and any weight that it carries, along with the mass of the air inside.)

Better than hot air are light gases such as hydrogen and helium. Air weighs about 1.25 kg per cubic meter. The same volume of hydrogen weighs about 14 times less, i.e. about 0.089 kg = 89 grams.[26] If we fill a one-cubic-meter balloon with hydrogen, it will tend to float. In fact, the "lift" (upward force) of the balloon is just equal to the difference in weights of air – hydrogen. Putting this into numbers, it means that a cubic meter balloon will have a lift of 1.25 kg – 0.089 kg = 1.16 kg. That means that if you hang an object from it that weighs less than one kilogram (including the mass of the balloon skin), it will still go upwards.

Helium gas isn't quite as light as hydrogen gas, so if the balloon is filled with helium, the lift isn't quite as much. Here's the calculation: helium gas weighs about 0.178 kg per cubic meter. So the lift on a helium balloon would be 1.25 kg – 0.178 kg = 1.07 kg. Note that even though the helium is twice as dense as hydrogen, the lift is almost as good.

[26] Nitrogen has an atomic weight (number of neutrons plus protons) of 14. Hydrogen has an atomic weight of 1. The number of atoms in a cubic meter is the same for both gases, so the factor of 14 simply reflects the larger nitrogen nucleus.

But 1.16 kg of lift for a cubic meter of balloon is not really much. That's why, despite the cartoons you may have watched on TV, even a large packet of balloons are not enough to lift a 25 kg child.

Hot air balloons have even less lift. If you heat the air to 300 °C, then it's temperature in absolute scale is 600 K. That's twice its normal temperature, so its density is half of its usual density. The lift of a cubic meter would be 1.25 kg – 1.25 kg /2 = 0.62 kg per cubic meter. Notice that this is significantly worse than hydrogen or helium. To lift a person who weighs 100 kg (including basket, balloon skin, cables) would take 100/0.62 = 161 cubic meters of hot air. If the balloon were shaped like a cube, the side of such a balloon would be $\sqrt[3]{161}$ = 5.4 meters = 18 feet. That's why hot air balloons have to be so big, and why they don't lift much.

Floating on Water

The same principles apply to floating on water. Salt water is denser than fresh water, so the density difference between it and you is greater; that's why you float higher in salt water. Even strong swimmers take advantage of their buoyancy (the fact that they are less dense, on average, than water). If water becomes filled with bubbles, its average density can become less than yours, and you will sink. The New York Times on August 14, 2003, had an article describing how a group of boys drowned because they were in bubbly water.[27] Undersea volcanic eruptions have led to bubbly water in the oceans, and in such water even ships will sink.

Submarines can adjust their depth below the ocean surface by taking in and expelling water into their ballast tanks. When they take in water, the air in the tanks is replaced by the heavier water, and the average density of the submarine increases. This makes the submarine sink. The only thing that will stop the sinking is expulsion of water from the tanks. If the submarine sinks too far, then it is crushed by the weight of the water above it; that makes it even denser, and so it sinks faster. That is called the hull crush depth. In

[27] Here is a quote from that article:

To four teenagers from the suburbs, Split Rock Falls was a magical place — cool water rushing between the granite walls of a mountain ravine, forming pools for hours of lazy summertime swimming.

On Tuesday afternoon the four men — Adam Cohen, 19; Jonah Richman, 18; Jordan Satin, 19; and David Altschuler, 18 — returned to their favorite childhood summer haunt to find it engorged by a summer of heavy rain. By the end of the day, all four men, each an experienced swimmer, were dead, drowned in the waters they knew well.

In what officials here described as one of the worst drowning accidents ever in the Adirondack State Park, all four died after Mr. Altschuler slipped off a narrow granite ledge into a foaming pool of water whipped into a frenzy by a tumbling waterfall. In a final act of friendship, Mr. Richman, Mr. Cohen and Mr. Satin, who had grown up together on Long Island, jumped after him to try to save his life, police and officials said. The laws of physics were against them, though.

"They call it a drowning machine," said Lt. Fred J. Larow, a forest ranger with the State Department of Environmental Conservation, who helped recover the bodies here, about 20 miles east of Lake Placid. "The water was so turbulent and aerated that there was no way they could stay above water. Even the strongest swimmer in the world couldn't have survived it."

the movie "Crimson Tide," that depth was 1800 feet, about 1/3 mile down.
The submarine in that movie (the fictional USS Alabama) was able to save itself by getting its engine running, and using its short "wings" to get lift in way similar to the way that an airplane does, by moving forward and pushing water downward. A submarine can also push compressed air into its ballast tanks, driving out the water, and decreasing its average density.

Sperm whales are said to be able to dive as deep at 2 miles. Diving deep is easy, since as the whale goes deeper, any air in its lungs (remember, a whale is a mammal) is compressed, and that makes the whale less buoyant. So once the whale is denser than water, it will sink. Coming up is the hard part. Whales save enough energy in their effortless dives, to be able to swim back up to the surface.

Air pressure on mountains, outside airplanes and satellites

Air pressure is simply the weight of the air above you. In any fluid or gas, the pressure is evenly distributed, so that the air at sea level will push an equal amount up, down, and sideways. As you go higher, there is less air above you, so the pressure decreases. At an altitude of 18,000 feet (3.4 miles, 5.5 km) the pressure is half of what it is at sea level. Go up another 18,000 feet – to 36,000 feet, and the pressure is reduced by another factor of 2 – to one quarter the pressure at sea level. That's a typical altitude for jet airplanes. And that rule continues; for every additional 18000 feet, the pressure (and the density of the air[28]) drops by another factor of two. You cannot live at such a low density of air, and that is why airplanes are pressurized. You can live at that pressure if the "air" you are given is pure oxygen (rather than 20%), and that is what you would get from the emergency face masks that drop down in airline seats if the cabin ever loses pressure.

The equation for this can be written in a simple way. Let P be the relative pressure at an altitude H, compared to the pressure at sea level. So for $H = 0$, $P = 1$. Then the equation is:

$$P = \left(\tfrac{1}{2}\right)^{H/18000ft} = \left(\tfrac{1}{2}\right)^{H/5.5km}$$

To work this out, take the altitude (say in km), divide it by 5.5, and write down your answer. Then multiply 1/2 by itself that many times. That can easily be done on a calculator. For example, consider a "low earth orbit" satellite, at 200 km above sea level. H/5.5km = 200/5.5 = 36.36 ≈ 36. Now multiply 1/2 by itself 36 times to give:

$$P = \left(\tfrac{1}{2}\right)^{36} = 1.45 \times 10^{-11} = 0.00000000001$$

[28] The air density doesn't drop quite that much because the air up there is cooler.

That's a pressure of 10 trillionths times as small as at sea level.[29] Satellites need this low pressure to avoid being slowed down by collisions with air.

Because the density of air decreases with altitude, a helium balloon will not rise forever. Eventually it reaches an altitude at which the outside air has the same density as the helium (with the weight of the gondola averaged in), and then it stops rising. That's why balloons are not a possible way to get to space.

Convection – thunderstorms and heaters

When air is heated near the ground, its density is reduced, and it tends to rise, just like a hot air balloon. Unconstrained by any balloon, the air expands as it rises, and an interesting result is that it will remain less dense than the surrounding air until it reaches the "tropopause", the level in which ozone absorbs sunlight causing the surrounding air to be warmer. When it reaches the tropopause, its density is no longer less than that of the surrounding air, so the air stops rising.

On a summer day, when thunderstorms are growing, it is easy to spot the tropopause. It is the altitude at which the thunderstorms stop rising, and begin to spread out laterally. The tropopause is a very important layer in the atmosphere. It is the location of the "ozone layer" which protects us from cancer-producing ultraviolet light. We'll talk more about this layer in Chapter 8 Waves (because of the important effect it has on sound), and in Chapter 10 Invisible light, when we discuss ultraviolet radiation and its effects.

Convection is the name we give to the process of hot air expanding and rising. When you have a heater in a room near the floor, the rising hot air forces other air out of the way; that results in a circulation of the air. This is a very effective way to warm a room – much faster than heat conduction through the air. But it invariably results in the warmest air being near the top of the room. If you put the heater near the top, the warm air just stays up there. On a cold day, in a room with a heater near the floor, stand up on a stepladder and feel how much hotter it is near the ceiling.

Angular Momentum and Torque

In addition to ordinary momentum (mass times velocity), there is another kind of momentum that physicists and engineers find enormously useful in their calculations called *angular momentum*. Angular momentum is similar to ordinary momentum but it is most useful for motion that is circular, i.e. rotation. If an object of mass M is spinning in a circle of radius R, moving at velocity v, then its angular momentum L is:

[29] The equation implicitly assumed that the temperature of the air continues to cool at higher and higher altitude. But above the tropopause the temperature rises, and that makes this calculation interesting, but not really accurate.

$$L = M v R$$

What makes angular momentum so useful is that, like ordinary momentum, when there are no external forces on an object, it is “conserved”, i.e. its value doesn’t change. Have you ever spun on ice-skates? Actually, ice skates are not necessary – just stand in a spot and start spinning with your arms stretched out. Then rapidly pull your arms in. To the surprise of most people, they will suddenly spin much faster. You can predict that from the angular momentum equation. If the angular momentum L is the same before and after the arms are pulled in, and the mass of the arms M is the same, then $v R$ must be the same. If R gets smaller, then v must get bigger.

Angular momentum conservation also explains why water leaving a tub through a narrow drain begins to spin. In fact, it is very unlikely that the water in the tub wasn’t already spinning, at least a little bit. But when the distance to the drain (R, in the equation) gets small, the v in the equation gets very big. A similar effect occurs in hurricanes and tornadoes. Air being sucked into the center (where there is a low pressure due to air moving upward) spins faster and faster, and that’s what gives the high velocities of the air in these storms. The air in hurricanes gets its initial spin from the spin of the Earth, and then they amplify it by the angular momentum effect. So hurricanes really do spin in different directions in the Southern and Northern Hemispheres.

It is not true that sinks or tubs drain differently in the northern and southern hemispheres. Their direction depends on the small residual rotation in the water left over from the filling of the tub, or from a person getting out.

Conservation of angular momentum can be used to understand how a cat, dropped from an upside down position, can still land on his feet. If he spins his legs in a circle, his body will move in the opposite direction, keeping his total angular momentum equal to zero. That way he can bring its legs underneath him. Astronauts do this trick when they want to reorient themselves in space. Spin an arm in a circle, and your body will move in the opposite direction. You can try that trick on ice skates too.

The conservation of angular momentum has other useful applications. It helps keep a bicycle wheel from falling over, when the wheel is spinning. It can also cause a difficulty: if kinetic energy is stored in a spinning wheel (usually called a “flywheel”), then the angular momentum makes it difficult to change the direction that the wheel is spinning. This makes its use for energy storage in moving vehicles, such as buses, problematical. It is often addressed by having two flywheels spinning in opposite directions, so although energy is stored, the total angular momentum is zero.

Angular momentum can be changed by a suitable application of an outside force. The required geometry is that the force must act obliquely, at a distance. We define *torque* as the tangential component of the force (the oblique part) times the distance to the center. So, for example, to start a bicycle wheel spinning, you can’t just push on the rim in a radial manner. You have to push tangentially. That’s called torque. The law relating torque and angular momentum is this: *the rate of change of angular momentum is* *numerically equal to the torque*. (Similarly, *force* may be defined as *the rate of change of (linear) momentum.*)

You can probably see why mastery of the equations of angular momentum is very useful to engineers and physicists in simplifying their calculations.

Quick Review

Weight is the force of gravity acting on your mass. The force of gravity is an inverse square law, so when the distance increases by (for example) 3, the force becomes 9 times weaker. It is this force that keeps the Moon in orbit around the Earth, and the Earth in orbit around the Sun. If gravity were turned off, satellites would move in straight lines rather than in circles. Even at great distances, the force of gravity never goes completely to zero. The sensation of weightlessness felt by astronauts is really the sensation of continuous falling.

All satellites must keep moving, or they fall to Earth; they cannot hover. In low Earth orbit (LEO), the satellite moves at 8 km/sec, and orbits the Earth in 1.5 hours. LEO satellites are useful for Earth observations, including spying. Geosynchronous satellites are useful for applications where the position of the satellite must stay fixed with respect to the ground. A medium Earth orbit (MEO) is in between. GPS satellites are MEO. A GPS receiver determines its location by measuring the distance to three or more of these satellites.

Close to the Earth (where we live) objects fall with constant acceleration *g*, i.e. their velocity is $v = gt$, until they are slowed down by air resistance. The distance they fall is $h = \frac{1}{2} g t^2$. Air resistance increases with the square of the velocity. If you go 3 times faster, the force of air resistance gets 9 times larger. Air resistance limits the velocity of fall to no more than the "terminal velocity." This depends on the area and mass of the object, and is different for people (60-100 mph), parachutes (15 mph), and large objects such as King Kong. Air resistance also limits the fuel efficiency of automobiles. Satellites must fly high (> 200 km) to avoid air resistance.

A force on an object makes it accelerate by an amount (given in m/s per sec) of

$$F = ma$$

This is known as "Newton's Second Law." Acceleration can also be measured in units of *g*, the acceleration of gravity. The g-rule says that to accelerate an object to 10 *g* requires a force 10 times as great as that object's weight. That's about the limit for a human to endure, so higher accelerations (e.g. the rail gun) can't be used. The Space Shuttle never accelerates more than 3 *g*. Circular motion can also be considered to be acceleration, with Newton's Second Law applying, even if the magnitude of the speed doesn't change. Based on this, we can calculate the velocity a satellite must have to stay in a circular orbit at different altitudes above the Earth.

The surface gravity on other planets and on asteroids is very different than it is on the Earth, science fiction movies notwithstanding.

The energy it takes to push an object with a force *F* (measured in Newton) over a distance *D* is

$$E = F\,D\,.$$

To escape to space completely requires an energy of about 15 Calories per gram. This is enough energy to lift you up a big enough elevator, if one could be built ("skyhook"). If you have a velocity of 11.2 km/sec, then your kinetic energy is sufficient to escape. Black holes are objects whose escape velocity exceeds the speed of light.

Gravity measurements have practical applications. Since oil is lighter than rock, it has a weaker gravity, and that fact has been used to locate it. Gravity measurements give us the best image of the Chicxulub crater.

When a gun is fired, the bullet goes forward and the gun goes backward. This is an example of the conservation of momentum. Other examples: Rockets go forward (very inefficiently) by shooting burnt fuel backwards. Airplanes and helicopters fly by pushing air downwards.

Objects float when their density is less than that of the fluid or gas they are in. That includes boats and balloons. Hot air rises because it is less dense than the surrounding air, and that happens for hot air balloons and thunderstorms. The density and pressure of air decreases with altitude according to a halving rule:

$$P = \left(\tfrac{1}{2}\right)^{H/5.5km}\,.$$

Angular momentum (a momentum that applies to circular motion) is also conserved, and that causes contracting objects to speed up. Examples include sink drains, hurricanes, and tornadoes.