Low Q Electrically Small Linear and Elliptical Polarized Spherical Dipole Antennas

Steven R. Best, *Senior Member, IEEE*

***Abstract*—Electrically small antennas are generally presumed to exhibit high impedance mismatch (high VSWR), low efficiency, high quality factor (Q); and, therefore, narrow operating bandwidth. For an electric or magnetic dipole antenna, there is a fundamental lower bound for the quality factor that is determined as a function of the antenna's occupied physical volume. In this paper, the quality factor of a resonant, electrically small electric dipole is minimized by allowing the antenna geometry to utilize the occupied spherical volume to the greatest extent possible. A self-resonant, electrically small electric dipole antenna is presented that exhibits an impedance near 50 Ohms, an efficiency in excess of 95% and a quality factor that is within 1.5 times the fundamental lower bound at a value of ka less than 0.27. Through an arrangement of the antenna's wire geometry, the electrically small dipole's polarization is converted from linear to elliptical (with an axial ratio of 3 dB), resulting in a further reduction in the quality factor. The elliptically polarized, electrically small antenna exhibits an impedance near 50 Ohms, an efficiency in excess of 95% and it has an omnidirectional, figure-eight radiation pattern.**

***Index Terms*—Dipole antennas, electrically small antennas, helical antennas, Q factor.**

I. INTRODUCTION

IN THEIR early work on electrically small antennas, Wheeler and Chu equated the small electric dipole to a lossy capacitor having a given effective height and effective cylindrical area. The input or feed point radiation resistance of the small dipole antenna is proportional to the square of its effective length [$\propto (l/\lambda)^2$] while its quality factor is inversely proportional to the cube of its effective volume [1], [2]. For small dipole antennas of the same effective length, an increase in the effective cylindrical area decreases the antenna's quality factor [3].

In this paper, a dipole is considered to be electrically small when $ka \le 0.5$, where k is $2\pi/\lambda$ and a is the radius of an imaginary sphere circumscribing the maximum dimension of the dipole [4]. The quality factor of a tuned antenna is defined by the ratio of reactive energy to accepted power (power loss within the antenna plus radiated power) [5]

$$Q(\omega_0) = \frac{\omega_0 W(\omega_0)}{P_A(\omega_0)} \tag{1}$$

where ω_0 is the tuned radian frequency. The tuned antenna's quality factor can also be determined directly from the antenna's untuned feed point impedance [6]

$$Q(\omega_0) = \frac{\omega_0}{2R(\omega_0)}\sqrt{R'(\omega_0)^2 + \left(X'(\omega_0) + \frac{|X(\omega_0)|}{\omega_0}\right)^2} \tag{2}$$

where $R(\omega)$ and $X(\omega)$ are the frequency dependent feed point resistance and reactance, respectively, and $R'(\omega)$ and $X'(\omega)$ are their frequency derivatives. This expression for antenna quality factor has been shown to be valid for both large and small ($Q < 2$) values of Q, for both electrically small and large antennas, and it has been shown to be inversely proportional to the antenna's matched VSWR bandwidth over wide ranges of frequency including ranges of both resonance and antiresonance [6], [7]. The antenna's quality factor and effective volume are related by [1]

$$r = \frac{\lambda}{2\pi}\left(\frac{9}{2Q}\right)^{1/3} \tag{3}$$

where r is the radius of a sphere defining the effective volume of the antenna. The lower bound on the quality factor of an electric or magnetic dipole antenna is given by [8]

$$Q_{lb} = \eta\left(\frac{1}{(ka)^3} + \frac{1}{ka}\right) \tag{4}$$

where η is the frequency dependent radiation efficiency of the antenna.

We begin by considering a small straight-wire dipole where $ka = 0.263$. This antenna exhibits a low feed point resistance, a high feed point capacitive reactance and a high Q. The capacitive reactance of the small straight-wire dipole can be tuned to zero using a number of techniques such as reactive loading at the feed point, capacitive top-loading or inductive loading within the antenna structure. Here, the reactance of the antenna is tuned to zero by increasing the effective self-inductance at the feed point through an increase in total wire length, while maintaining a fixed overall length. The feed point resistance of the antenna can be transformed to 50 Ohms using a number of techniques including impedance matching and the use of multiple folded arms. Both techniques are discussed here with the final electrically small dipole design being impedance matched using multiple folded arms. The quality factor of the antenna is minimized by utilizing the occupied spherical volume (while maintaining a fixed ka) to the greatest extent possible.

II. NONFOLDED SMALL DIPOLE ANTENNAS

Consider a straight-wire electric dipole with an overall length l equal to 47.3 cm and a wire diameter equal to 2.6 mm. The impedance properties of this dipole were simulated using the NEC4 engine of EZNEC Pro [9]. The dipole was modeled to include copper loss. This antenna is self-resonant at a frequency of approximately 300 MHz. This self-resonant straight-wire dipole has a Q equal to 6.1 and an r/λ equal to 0.144, indicating that the diameter of the dipole's effective spherical volume is less

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The author is with the Air Force Research Laboratory (AFRL/SNHA) Hanscom AFB, MA 01731 USA (e-mail: steven.best@comcast.net).

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than its overall length. The straight-wire dipole is self-resonant at a frequency where $ka = 1.487$.

Now consider an electrically small, straight-wire dipole with an overall length l equal to 8.36 cm and a wire diameter of 2.6 mm. At 300 MHz, this antenna has a feed point impedance equal to $1.1 - \mathrm{j}1015$ Ohms. The VSWR with respect to 50 Ohms is equal to 18 768:1. The small dipole's efficiency is 98%, its quality factor is 950, and r/λ is equal to .0268. ka for this electrically small dipole is 0.263. For $\eta = 0.98$ and $ka = 0.263$, the lower bound on Q is equal to 57.6. The quality factor of this electrically small dipole is 16.5 times the lower bound.

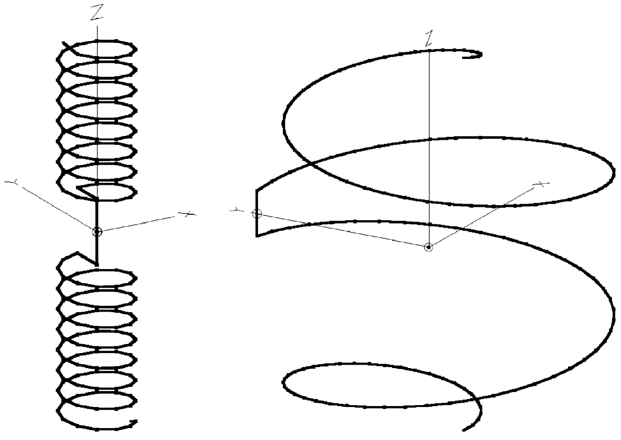
The quality factor of the electrically small antenna can be decreased by increasing the effective volume of the antenna. While this can be accomplished by simply increasing the size or physical volume occupied by the antenna (at some point the antenna will no longer be considered electrically small), the objective here is to increase the antenna's effective volume without increasing its occupied physical volume such that ka is held constant. In this case, the electrically small antenna is confined to an occupied physical volume defined by a sphere of radius a, such that $ka \leq 0.263$ at 300 MHz. The maximum overall length of the electrically small antenna is less than or equal to 8.36 cm.

The electrically small antenna exhibits maximum effective volume (expressed as r/l) at its self-resonant frequency [4]. Self-resonance is achieved by simply increasing the total wire length within the defined spherical volume. The increase in total wire length can be implemented with any choice of geometry. Here, a normal mode helix geometry, depicted in Fig. 1, is chosen. This antenna has an overall length of 8.36 cm and a total wire length of 78.65 cm. The wire diameter of this and the other antennas considered here is 2.6 mm.

The normal mode helix dipole is self-resonant at 299.6 MHz with a resonant resistance (including copper loss) of approximately 4.6 Ohms. The antenna's Q is 216.6 and r/λ is equal to 0.0438. The quality factor has decreased by a factor of approximately 4.4 relative to the quality factor of the electrically small straight-wire electric dipole. Note also that the resonant resistance has increased by a factor of approximately 4.2.

Given that the quality factor is inversely proportional to resistance, one might conclude that the decrease in quality factor can be directly attributed to the increase in resistance. This is not the case and the similarity in the increase in resistance and the decrease in quality factor is coincidental. In addition to being a function of the antenna's resistance, the quality is also a function of the frequency derivatives of both resistance and reactance. For the electrically small self-resonant antenna, $|X'|$ is generally much greater than $|R'|$ and it is the dominant factor in establishing the antenna's Q. As the occupied volume of the antenna increases, while ka remains fixed, the reactive fields within the antenna volume decrease resulting in a decrease in Q, which is fundamentally defined by the ratio of reactive energy to accepted power.

Consider the spherical helix dipole antenna also depicted in Fig. 1. This antenna has an overall length of 8.22 cm and fits within a spherical volume defined by $a = 4.15$ cm. The spherical helix dipole is resonant at 300.2 MHz with a resonant resistance of approximately 2.2 Ohms. The antenna's quality factor is 143.9 and r/λ is equal to 0.0501. Although the resonant resistance of the spherical helix is less than that of the normal mode helix, its quality factor is substantially lower. This can be attributed to the fact that the spherical helix occupies a substantially larger physical volume (with the same ka) and therefore exhibits a greater effective volume. The reactive fields within the volume surrounding the spherical helix are substantially less than the reactive fields surrounding the normal mode helix.



Normal Mode Helix Spherical Helix

Fig. 1. Depictions of the electrically small normal mode helix and spherical helix electric dipole antennas. Both are self-resonant near 300 MHz, where $ka \approx 0.27$.

Another point to note is that the total wire length in the spherical helix is 58.23 cm, which is significantly less than that of the normal mode helix. Wire geometries that occupy a greater physical volume and more specifically, are such that the wire is confined to lie along the perimeter of the occupied volume, generally require less total wire length to achieve self-resonance [3].

III. FOLDED SPHERICAL HELIX DIPOLE ANTENNAS

In addition to decreasing the quality factor of the electrically small antenna, another design goal is to increase the resonant resistance to a value close to 50 Ohms so that a reasonable impedance match is achieved. An increase in resonant resistance can be realized by adding folded arms to the spherical helix dipole [10], as illustrated by the 2-arm folded spherical helix dipole depicted in Fig. 2.

The 2-arm folded spherical helix dipole is created by duplicating the arms in the spherical dipole depicted in Fig. 1 and rotating them 180 degrees. The arms are then connected at the top and bottom of the dipole. The antenna has a single feed point at the center of one of the vertical sections in the antenna. The spherical dipole depicted in Fig. 2 has an overall length of 8.25 cm and fits within a spherical radius defined by $a = 4.15$ cm. The total wire length in one dipole arm is 62.8 cm. The antenna is self-resonant at 299.6 MHz with a resonant resistance of approximately 10.3 Ohms. The antenna's quality factor is 101.1 and r/λ is equal to 0.0564. The addition of another set of connected dipole arms within the antenna's occupied volume decreases the total reactive fields within the volume surrounding the antenna, lowering the antenna's Q.

A further increase in radiation resistance and decrease in quality factor can be accomplished with the further addition of folded arms as illustrated with the 4-arm folded spherical dipole depicted in Fig. 3. This antenna has a single feed point

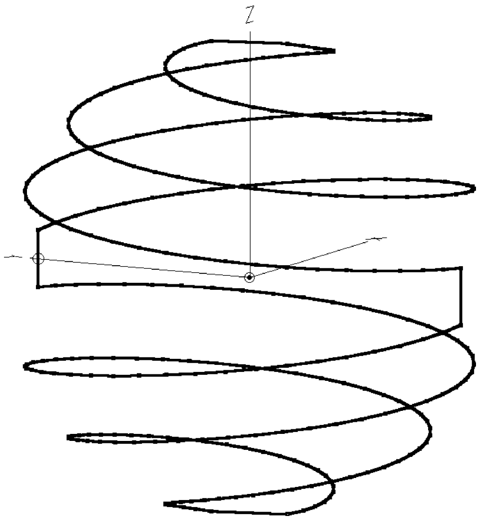


Fig. 2. Depiction of the electrically small, 2-arm folded spherical helix dipole antenna. The antenna has overall length of 8.25 cm and is resonant at 299.6 MHz with a resonant resistance of 10.3 Ohms.

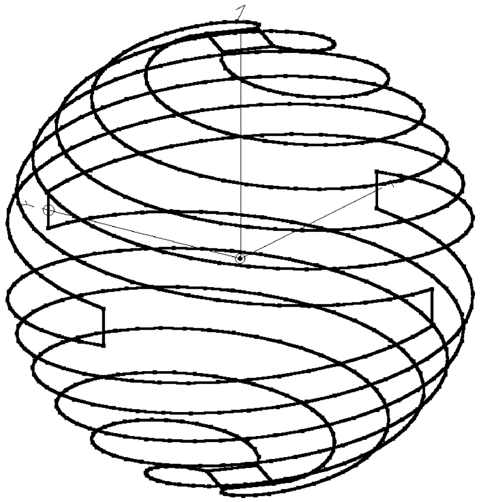


Fig. 3. Depiction of the electrically small 4-arm folded spherical helix dipole antenna. The antenna has overall length of 8.36 cm and is resonant at 299.9 MHz with a resonant resistance of 47.5 Ohms.

in one of the vertical arms at the center point in the antenna's overall length. This antenna has an overall length of 8.36 cm and fits within a spherical radius defined by $a = 4.18\ \mathrm{cm}$. The total wire length in one dipole arm is 65.53 cm. The antenna is self-resonant at 299.9 MHz with a resonant resistance of approximately 47.5 Ohms. The antenna's quality factor is 87.3 and r/λ is equal to 0.0592. The 4-arm spherical helix dipole has an efficiency of approximately 97.4%. Using (4), the lower bound on Q is determined to be approximately 57.3. The quality factor of the 4-arm folded spherical helix dipole is within 1.52 times the lower bound on Q for an electric dipole at $ka \approx 0.263$.

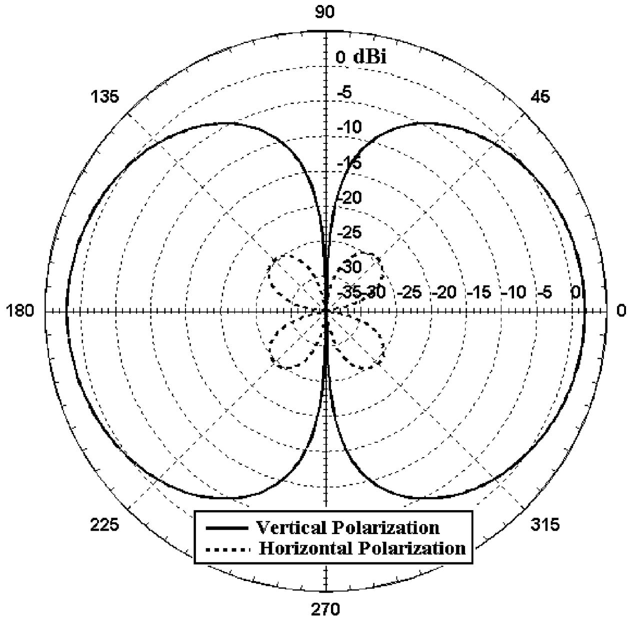


Fig. 4. Radiation pattern of the 4-arm folded spherical helix dipole at 299.9 MHz. The antenna exhibits an omnidirectional, linearly polarized radiation pattern consistent with that of an electric dipole. The antenna has a gain of 1.9 dBi.

TABLE I
COMPARISON OF THE ANTENNAS' RESONANT PROPERTIES

Antenna	Resonant Frequency (MHz)	Resonant Resistance (Ohms)	*Q*	*r/λ*
Straight-Wire Dipole	300*	1.1 − j1015	950	0.0268
Normal Mode Helix	299.6	4.6	216.6	0.0438
Spherical Helix	300.2	2.2	143.9	0.0501
2-Arm Folded Spherical Helix	299.6	10.3	101.1	.0564
4-Arm Folded Spherical Helix	299.9	47.5	87.3	0.0592

* The small straight-wire dipole is not resonant near 300 MHz. Its performance at 300 MHz is presented for a relative comparison.

The antennas depicted in Figs. 1–3 are equivalent to electric dipoles because they exhibit a single resonance consistent with a series *RLC* circuit and they exhibit a vertically polarized, omnidirectional radiation pattern consistent with that of a straight-wire dipole antenna. The horizontal polarized component of radiation (directivity) is significantly diminished because the horizontal components of current within the wire geometry effectively cancel. The radiation pattern of the 4-arm folded spherical dipole is depicted in Fig. 4. The gain of the antenna (including the affects of copper loss) is 1.9 dBi. A comparison of the antennas' resonant properties is presented in Table I. Graphical comparisons of the antennas' quality factors and r/λ are depicted in Figs. 5 and 6, respectively.

IV. THE ELLIPTICALLY POLARIZED SMALL SPHERICAL DIPOLE

The 4-arm folded spherical helix antenna depicted in Fig. 3 exhibits linear, vertical polarization. With a rearrangement of wire geometry within the lower dipole arms relative to the upper dipole arms, the horizontal component of current can be made to add constructively, reenforcing the radiation of linear, horizontal polarization [12]. Pure circular polarization (0 dB axial ratio) is

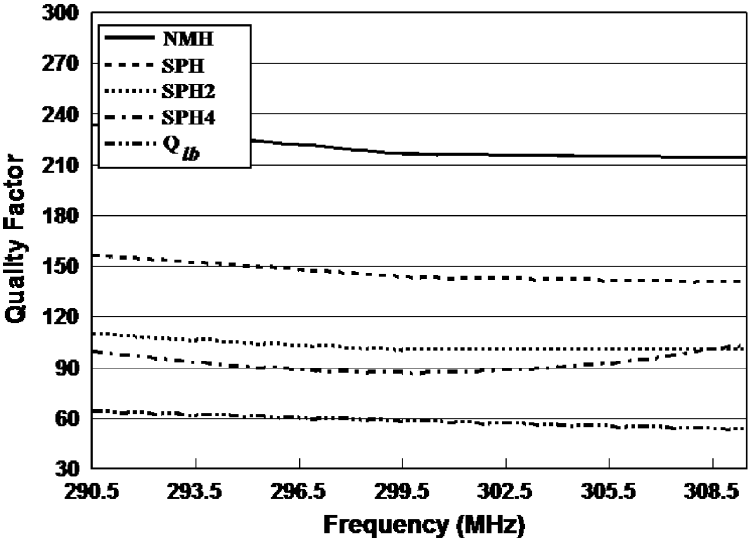


Fig. 5. Comparison of the quality factors of the antennas depicted in Figs. 1–3. The lower bound on Q is also presented for reference.

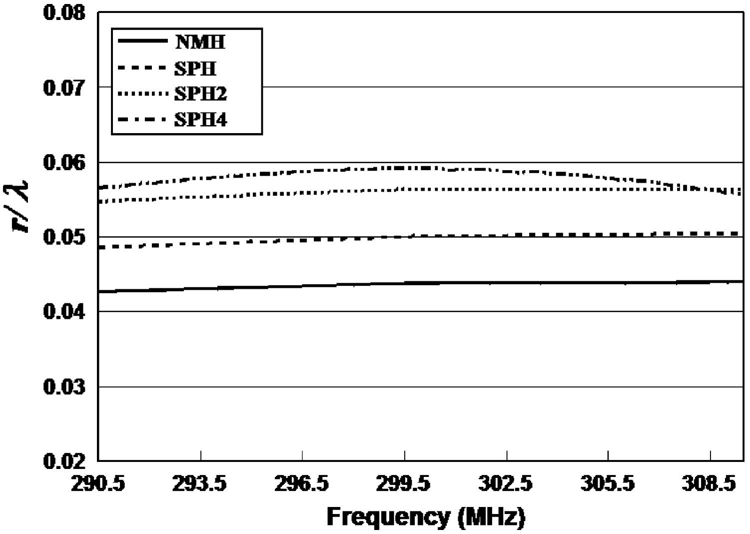


Fig. 6. Comparison of r/λ for the antennas depicted in Figs. 1–3.

achieved when the linear vertical and horizontal polarizations are of equal amplitude and are 90 degrees out-of-phase.

A rearrangement of wire that reenforces the radiation of horizontal polarization is depicted in Fig. 7. Additionally, the horizontal polarized radiation is 90 degrees out-of-phase with the vertical polarized radiation at all angles. The level of horizontal polarized radiation is approximately 3 dB less than the vertical polarized radiation, resulting in an axial ratio of 3 dB. With a 3 dB axial ratio, the antenna exhibits elliptical not circular polarization. The 3 dB axial ratio exhibited by the configuration of Fig. 7 is a result of the specific geometry and is not considered to be a theoretical limit for this design approach.

The radiation pattern of this antenna is depicted in Fig. 8. The antenna is resonant at a frequency of 301.3 MHz with a resonant resistance of 61.5 Ohms, which is slightly higher than that of the linear polarized configuration. However, the VSWR with respect to 50 Ohms is less than 1.5:1. The efficiency of the antenna is 98.7%. It has a calculated "circular" polarized gain of approximately 1.7 dBic.

The lower bound on the quality factor for a circular polarized antenna is less than that of a linear polarized antenna and is expressed as [8]

$$Q_{lbcp} = \eta \frac{1}{2}\left(\frac{1}{(ka)^3} + \frac{2}{ka}\right). \tag{5}$$

The quality factor of the elliptically polarized antenna configuration of Fig. 7 would be approximately half of the quality factor of the linear polarized antenna configuration depicted in Fig. 3 if its axial ratio were 0 dB, where the vertical and horizontal components each contain half the radiated power. With an axial ratio greater than 0 dB, the reduction in quality factor would be between half and 1 as a direct function of the ratio of power contained in the vertical and horizontal polarized components. The elliptical polarized antenna of Fig. 7 has an axial ratio of 3 dB and the vertical polarized component of radiation contains two-thirds of the total radiated power. With this ratio

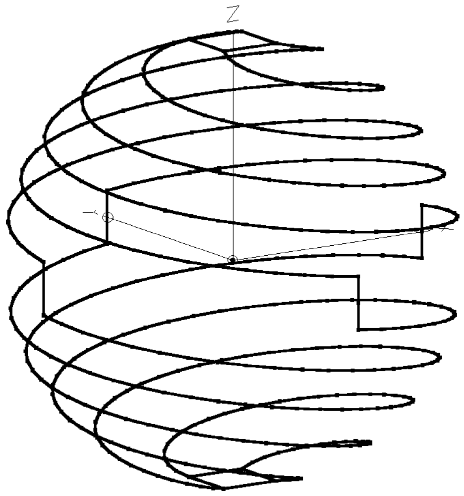


Fig. 7. Depiction of the elliptically polarized, electrically small, 4-arm folded spherical helix antenna. The antenna has an overall length of 8.36 cm and is resonant at 301.3 MHz with a resistance of 61.5 Ohms.

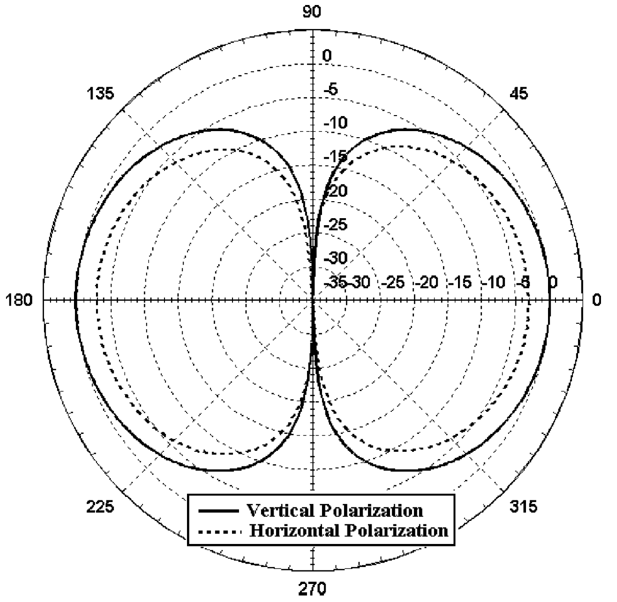


Fig. 8. Radiation pattern of the elliptically polarized 4-arm folded spherical helix dipole at 301.3 MHz. The antenna exhibits an omnidirectional, elliptically polarized radiation pattern with an axial ratio of 3 dB. The antenna has a gain of 1.7 dBic.

of power distribution between the vertical and horizontal polarization components it is expected that the quality factor of the elliptical polarized antenna would be reduced by approximately two-thirds (66.67%). Using (2), the quality factor of the elliptical polarized antenna of Fig. 7 is found to be 61.7, a 70.7% reduction. The lower bound on the quality factor for the elliptically polarized antenna calculated using (5) is approximately 31. The quality factor of the elliptically polarized antenna is approximately 2 times the lower bound at $ka \approx 0.263$.

V. IMPEDANCE MATCHED NONFOLDED SMALL ANTENNAS

The 4-arm folded spherical helix antennas depicted in Figs. 3 and 7 were impedance matched to 50 Ohms through the use of multiple folded arms. With these antenna configurations, the resonant frequency is established by the total length of wire in each of the four arms. There is a direct relationship between the arm length, the number of folded arms, the antenna's overall spherical radius and the values of resonant frequency and resonant resistance [10].

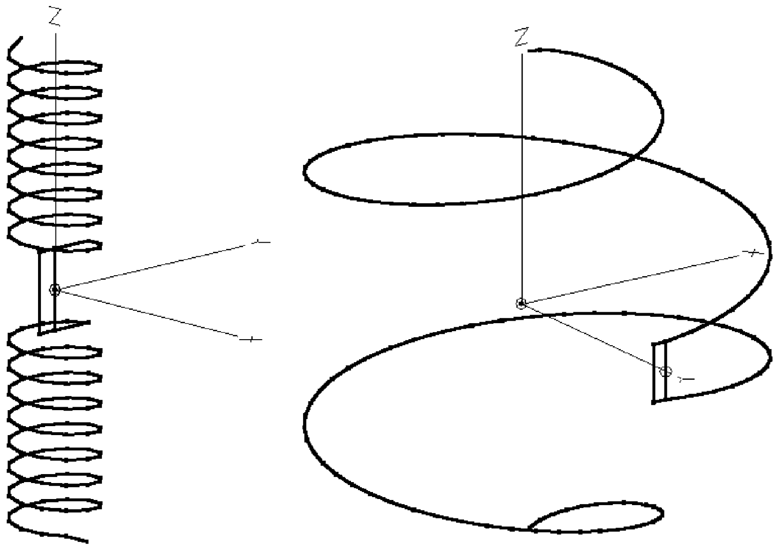
For the specific total wire length (65.53 cm), occupied spherical radius (4.18 cm), and the number of folded arms (4) in the antenna configuration of Fig. 3, resonance is achieved near 300 MHz, with a resistance near 50 Ohms. For a fixed number of folded arms, the total wire length and occupied spherical radius can be adjusted to achieve resonance over a very wide range of frequencies. However, the resonant resistance of the antenna will differ as a direct function of the antenna's overall length relative to the resonant wavelength, since the resonant resistance is proportional to $(l/\lambda)^2$. If the spherical radius of the 4-arm folded spherical helix is increased and the total wire length is adjusted to maintain a resonant frequency near 300 MHz, the resonant resistance will increase above 50 Ohms. If the spherical radius is decreased and the total wire length is adjusted to maintain resonance near 300 MHz, the resonant resistance will decrease below 50 Ohms. This limits the choice of physical antenna configurations while trying to maintain a 50-Ohm impedance at resonance.

As an alternative to the multiple folded arm configuration an electrically small, resonant single arm configuration, similar to those depicted in Fig. 1, with a resonant resistance substantially less than 50 Ohms, can be matched to 50 Ohms using a reactive matching network or a parallel-stub between the upper and lower dipole arms []. Parallel-stub impedance matched configurations of the normal mode helix and the nonfolded spherical helix are depicted in Fig. 9. The parallel-stub matched normal mode helix is matched at a frequency of 295.8 MHz with an impedance of $54.5 - \mathrm{j}4.2\ \mathrm{Ohms}$, while the parallel-stub matched spherical helix is matched at 295.1 MHz with an impedance of $52.2 - \mathrm{j}4.9\ \mathrm{Ohms}$.

While these parallel-stub matched configurations are impedance matched to 50 Ohms, there is not a substantial reduction in their quality factors. The Q of the normal mode helix configuration is reduced from 216.6 to 206.3, while the Q of the spherical helix configuration is reduced from 143.9 to 143.2. The multiple arm folded dipole configurations offer a substantially lower Q since the reactive fields within the volume of the antenna are substantially reduced.

VI. MEASURED DATA

Validation of the NEC simulations was performed by constructing a monopole (the upper portion of the antenna is mounted over a ground plane) version of the 4-arm, linear polarized folded spherical helix antenna [10]. The monopole version of the antenna was configured so that it was self-resonant near 300 MHz with a resonant resistance near 50 Ohms. This required the antenna height (over ground) to be increased to approximately 6 cm. A comparison of the measured and NEC simulated impedance over a wide range of frequencies is presented in Fig. 10. The NEC simulation accurately predicts the impedance properties of these antennas.



Stub Matched Normal Mode Helix

Stub Matched Spherical Helix

Fig. 9. Depictions of the parallel-stub matched normal mode helix and spherical helix antennas.

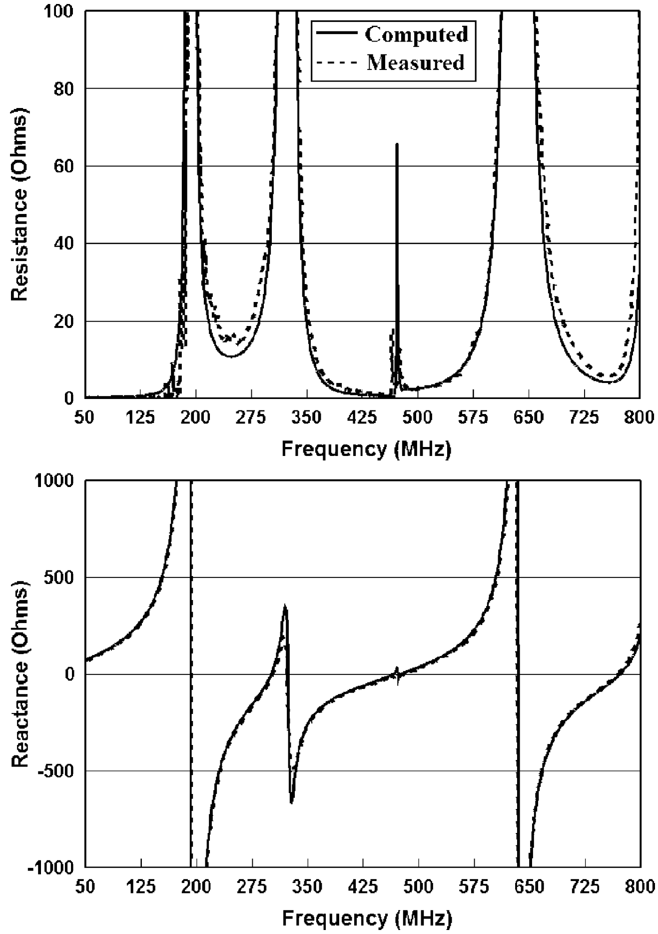


Fig. 10. Comparison of measured and simulated impedance for a 4-arm monopole version of the folded spherical helix antenna [10].

VII. Discussion

An electrically small dipole antenna can be made to achieve self-resonance through an increase in total wire length while maintaining a fixed height and a fixed occupied volume. The small dipole can be impedance matched to 50 Ohms using a number of techniques including reactive matching at the feed point, parallel-stub matching at the feed point, or through the use of multiple folded arms. The multiple folded arm configurations presented here are configured to take advantage of and occupy the full spherical volume for a fixed value of ka. The use of multiple folded arms in the spherical helix geometry reduces the total reactive fields within the volume of the antenna compared to the parallel-stub matched configurations, resulting in a lower Q.

The 4-arm folded linear polarized spherical dipole presented exhibits self resonance at 300 MHz, occupying a volume where $ka \approx 0.263$. It has a resonant resistance of 47.5 Ohms, an efficiency of 97.4% and a Q of 87.3, which is within 1.5 times the theoretical lower bound on Q for an electric dipole. Through a rearrangement of wire geometry, the horizontal currents within the antenna are configured to add constructively resulting in a significant increase in horizontally polarized radiation. This electrically small antenna is elliptically polarized with an axial ratio of 3 dB. It exhibits resonance near 300 MHz having a resistance of 61.5 Ohms, an efficiency in excess of 98% and a quality factor within two times the lower bound on quality factor for a circularly polarized antenna at a $ka \approx 0.262$.

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Steven R. Best (S'82–M'83–SM'98) was born in Saint John, NB, Canada. He received the B.Sc.Eng. and Ph.D. degrees in electrical engineering from the University of New Brunswick, in 1983 and 1988, respectively.

He has over 17 years of experience in business management and antenna design engineering in both military and commercial markets. In August 1987, he joined Chu Associates, Incorporated, El Cajon, CA, as a Senior Design Engineer where he worked on the design of numerous HF, VHF, and UHF antennas for government, military, and commercial applications. In 1990, he was appointed to the position of General Manager and in 1992, he was appointed to the position of Vice President and General Manager. In December 1993, he cofounded Parisi Antenna Systems, Waltham, MA. In June 1996, he joined Cushcraft Corporation, Manchester, NH, as Director of Engineering and was subsequently appointed to the position of company President in August 1997. He is currently with the Air Force Research Laboratory (AFRL/SNHA) at Hanscom AFB, MA, where his areas of interest include electrically small antennas, wideband radiating elements, conformal antennas, phased arrays, and communication antennas. He is the author or co-author of over 80 papers in various journal, conference and industry publications. He frequently presents a three-day short course on antennas and propagation for wireless communications and he is the author of a CD-ROM series on antennas for wireless communication systems.

Dr. Best is a member of Sigma Xi and a member of ACES. He is an Associate Editor for the IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS and a Vice-Chair for the IEEE Boston Section.