Topological Properties of Quantum States of Condensed Matter: some recent surprises.

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I. Berry phases, zero-field Hall effect, and “one-way light”

II. Anomalous and Spin Hall effect, Topological insulators

III. Non-abelian FQHE states
Bands with both time-reversal and spatial-inversion symmetry:

- Bands are doubly-degenerate at generic points in the Brillouin zone.

- Bands at special k-points where $2k_j = G$ are classified by inversion symmetry $I_i(k_j) = +1$ or -1 about inversion center $i$ in the real-space unit cell.

- In 2D (3D) there are 4 (8) special k-points and 4 (8) distinct inversion centers; the product over all bands below the Fermi energy is

$$\eta_0 = \prod_j I_i(k_j) = \pm 1$$

Fu and Kane, 2006

- In 2D and above this is independent of which inversion center $i$ is chosen.

This is clearly a “topological invariant”, but Kane and Fu’s recent (2006) result shows the not-so-obvious fact that it must have the value $+1$ in the absence of spin-orbit coupling.
"Quantum Spin-Hall effect"

Current-carrying state has a finite magnetization

\[ \mathbf{M} \propto I \]

- (But not quantized)

- On edge, there is a local magnetization direction

\[ \mathbf{B} \times \hat{n} \] produces current

\[ \mathbf{B} \perp \hat{n} \] backscatters

\[ (B \psi_k^+ \psi_L e^{i\phi} + h.c) \]
3D topological insulators

1) If inversion symmetry is present, (as well as time reversal symmetry), just count number of odd parity bands below Fermi level at points $\vec{K} = \vec{G}/2$ ($\vec{G}$ a reciprocal lattice vector)

- Bismuth (Semi metal)
- L (3)
- Antimony

Bose Sbz
$x \approx 0.1$
topological insulator
Reversal of order of bands with opposite Inversion symmetry
3D $\mathbb{Z}_2$ invariant (general case)

$R = (100)$

$k$-space plane includes $(000)$

$k$-space plane without $(000)$

$Y_+ Y_- = -1$ (however $\vec{R}$ is chosen)

Can calculate $Y_\pm$ from Berry phase integral over 2D $k$-space plane
on every crystal face, find a 2D metal

\[ \prod e^{i \phi^B_v} = e^{i \phi_{\text{total}}} \]

Berry phase for moving around Fermi surface

Karplus Luttinger: \[ \sigma_{\text{xy}} = \frac{e^2}{h} \left( \frac{\phi_{\text{total}}^B}{2\pi} + \text{Integer} \right) \]

Time reversal: \[ e^{i \phi_{\text{total}}^B} = e^{-i \phi_{\text{total}}^B} \]

\[ e^{i \phi_{\text{total}}^B} = \eta = -1 \]

topological insulator

= \frac{1}{2} + \text{Integer}
Localization is absent!

(a) \[ \begin{array}{c}
\text{I} \\
\text{II} \\
-k^\uparrow \\
-k^\downarrow \\
\end{array} \]

No SOC
Berry phase + 1 around Fermi surface which is \( k \leftrightarrow -k \) symmetric.

Time-reversed paths I and II from \( k^\uparrow \) to \( -k^\uparrow \) constructively interfere.

Extra 180° backscattering

\[ \Theta_{kk'} \rightarrow \text{Weak (Anderson)} \]

Localization

\[ \propto \frac{1}{d} \]

Amplitude
(b) Usual case with SOC (Symplectic)

- 2 spin-split Fermi surfaces
- Weak scattering allows only paths from $k^\uparrow$ to $-k^\downarrow$
- Berry phase $-1$
- Destructive interference

Amplitude

Conductance

\[ \ln g \]

\[ \ln D^{-1} \]

- Strong disorder allows scattering from $k^\uparrow$ to $-k'^\uparrow$ (no interference)
- Also interactions affect weak antilocalization
But, if the "other" spin-split Fermi surface is absent

\[ k^\uparrow \text{ and } -k^\uparrow \]
\[ \text{are on opposite faces of sample "sandwich"} \]

No destruction of weak antilocalization by strong disorder

Topological insulator.
Open questions

- Nature of conduction of interacting surface of topological insulator
- Magnetic/superconducting order
- Majorana fermions, proximity effect