

Far Field Calculation of a Lens-Corrected Corrugated Horn

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Purpose:

A description of the operation of a lens-corrected horn.

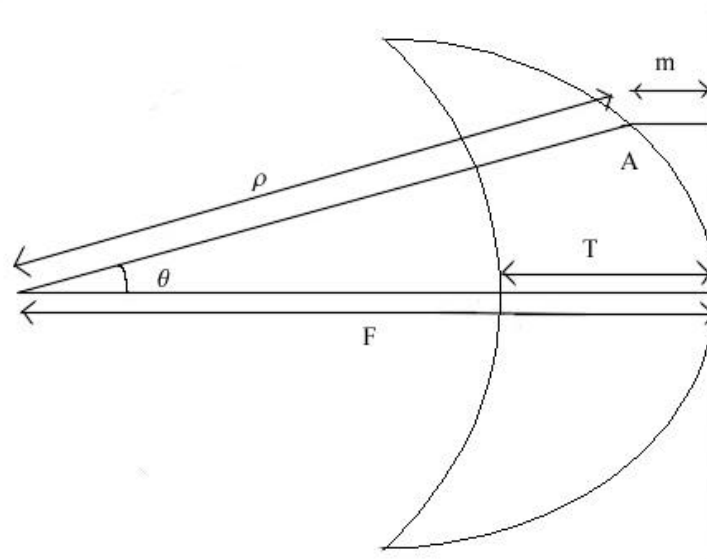
Advantages of using a Lens:

- ?? Control the radiation pattern (most important for us)
- ?? Achieve high aperture efficiency
- ?? Achieve good cross-polarization and voltage standing wave ratio (VSWR)
- ?? Achieve a constant phase center

Shape of Lens:

We want a constant phase across the aperture of the lens. Thus, the shape of the lens can be derived using Fermat's principle and equalizing all the optical distances from the horn to the aperture. This is only an approximation (the lens is most likely placed in the near field), but experience has shown that it provides acceptable results. We chose a lens with only one refractive surface to minimize the complications in building the lens. The surface facing the horn is a sphere that is locally perpendicular to the radiation and thus does not refract.

The shape of the outer surface is determined by equating the optical path through a point A on the elliptical surface to the optical path along the axis (See diagram).



$r = \frac{m}{\cos \theta} \left(1 - \frac{F}{m} \right)$ from geometry (11.4)
 $r = \frac{m}{\cos \theta} \left(1 - \frac{F}{m} \right)$ optical path equivalence (11.5)
 Solving (11.4) and (11.5) :

$$r = \frac{F}{\cos \theta} \left(1 - \frac{F}{m} \right)$$

This describes the shape of the outer surface of the lens in polar coordinates. It is an equation for an ellipse with the origin at one focus. Note that this formula (11.4) is incorrect in Olver.¹

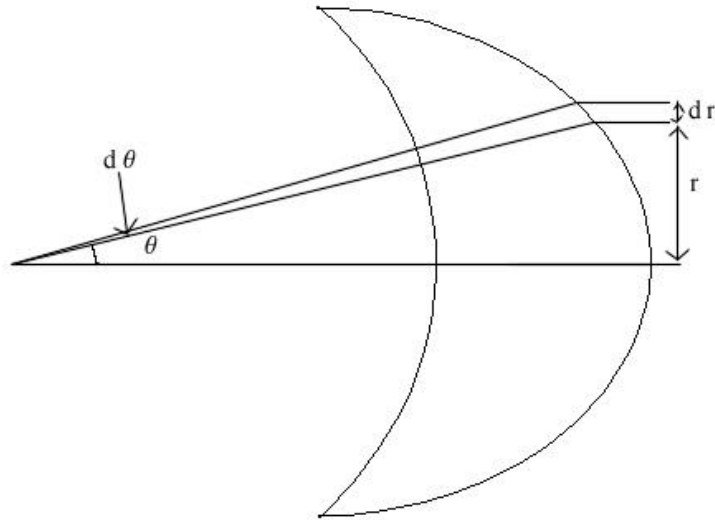
Note: Ch. 11 (Lens-Corrected Horns) in Olver does not recommend this type of lens because the angle (called theta max) from the edge of the lens when the lens is half an ellipse to the axis must always be greater than the semi-flare angle of the horn for the lens to be placed right on top of the horn aperture. (From pure geometry) This places a lower limit on theta max and thus places a higher limit on the index of refraction. We can still use such a lens without any restrictions if we do not place the lens on the horn aperture but instead position it further away. However, this introduces mounting problems, increases the size of the lens, and results in the loss of energy. For the future, it might be best to use a lens with a straight surface facing the horn, which can always be adjusted properly at the horn aperture, and the index of refraction can be varied to get the right radiation pattern.

Aperture Amplitude Distribution:

The power across the output aperture of the lens is influenced by four factors:

- ?? power at the surface of the lens
- ?? reflections at the surface of the lens
- ?? dielectric losses in the lenses
- ?? shape of the lens

The power at the surface of the lens is called P_s and will be discussed further in the next section. Reflections at the surface of the lens and dielectric losses are usually minimal and can be safely ignored in the approximation that we are working in. For example, a lens made of polystyrene with a thickness of one wavelength has an attenuation of 0.004 dB. Thus, the major factor affecting the amplitude distribution across the output aperture of the lens is the shape of the lens. Rays passing through the lens become parallel after the lens. Setting equal the power through an angular ring to the power through the output aperture ring:



$dP = 2 \sin \theta P r dr$? power in solid ring? ?1?

$dP = 2 r P r dr$? power in aperture ring? ?2?

Equating ?1? and ?2?: $P r = P r$? $\frac{\sin \theta dr}{r dr}$?3?

Also: $\frac{F \cos \theta}{n \cos \theta}$? equation of lens?

$r \cos \theta = \frac{F \cos \theta}{n \cos \theta}$? we can calculate $d\theta$? dr from here?

Substituting for $d\theta$? dr into ?3? :

$$P r = P r \frac{n \cos \theta}{F^2 \cos^3 \theta}$$

$P[r]$ is the power as a function of the distance from the lens axis (after the lens).

The Problem of $P[\theta]$:

$P[\theta]$ is the power across the input aperture as a function of the angle from the lens axis and is a characteristic of the horn used. We used three approaches to model $P[\theta]$.

?? Gaussian Approach:

We know the full width half max of $P[\theta]$ from previous experiments. We can model a gaussian having the same full width half max to serve as the $P[\theta]$ in the program.

FWHM is the Full Width Half Max of the field pattern

$$?? ? \frac{FWHM^2}{8 \ln 2}$$

$$P[\theta] = e^{-\frac{\theta^2}{2}}$$

?? Spherical Wave Expansion:

The Spherical Wave Expansion is a technique to derive the closed form expression for the field pattern of the horn. ² We created a Mathematica worksheet to implement this technique. Unfortunately, the results of this worksheet were not in agreement with theory. (See Appendix I for the Mathematica Worksheet)

?? Dadra and Curve Fitting:

Dadra takes the spherical wave coefficients from ccrhn and produces the field pattern at the distance required. We used Dadra to get the field pattern at various distances and then used a curve-fitting program³ to fit a 10 degree polynomial to the field pattern to obtain P[?]. Since the field pattern is an even function we omitted all the odd power terms. (See Appendix II for the various fits)

Kirchoff-Huygens Formula:

Although geometric optics says the rays stay parallel after passing through the lens, in reality they begin to diverge immediately after. Given P[r], the Kirchoff-Huygens formula approximates the far field pattern of an aperture using wave optics.

$$f(u) \approx \frac{D^2}{2} \int_0^{D/2} P(r) J_0(ur) r dr$$

where $r = \frac{r}{D/2}$ and $u = \frac{D}{\lambda} \sin \theta$

Note: The Kirchoff-Huygens formula (11.25) in Olver is incorrect. Also note that we have substituted θ for θ' in the expression for u because it is different from the θ' used previously.
 θ = angle from the lens axis used to define the lens surface
 θ' = angle from the lens axis used to define the far field
 (both are centered at the phase center of the horn)

Variables used in Mathematica Notebook:

- F = focal length of lens
- r_{min} = radius of curvature of lens back
- n = index of refraction of lens material
- r_{max} = half - angle of lens
(determined by choosing an attenuation point, usually 20 or 30 dB, and reading off the corresponding angle from Dadra's output file)
- λ = wavelength
- dia = diameter of lens
- θ = angle from the lens axis (at the lens face)
- φ = angle from the lens axis (at the far field)
- r = distance from the lens axis
- P₀[r] = power (before the lens) as a function of r
- P_r[r] = power (after the lens) as a function of r
- θ[r] = θ as a function of r
(determined by the shape of the elliptical face)
- stepsize = determines the accuracy of the results

The distance from the phase center to the refracting elliptical surface (F) is determined by

$$F = \frac{r_{min} \sin \theta_{max}}{n - 1}$$

θ[r] can be calculated by either solving the transcendental equation $r = \frac{F(n-1)}{n \cos \theta}$ for θ or by finding the Cartesian equation of the elliptical face. We tried both methods and they give equivalent results.

Transcendental Approach:

$$r = F \cdot \text{ArcCos} \left[\frac{n-1}{n} \cos \theta \right] = F \left[1 - \frac{n^3}{4F^4} n^2 + \frac{6F^4}{4F^4} n^3 - \frac{F^4}{n^4} + F^2 r^2 - 2F^2 n r^2 + 2F^2 n^3 r^2 - F^2 n^4 r^2 \right] = F \left[1 - \frac{n^3}{4F^4} n^2 + \frac{6F^4}{4F^4} n^3 - \frac{F^4}{n^4} + F^2 r^2 - 2F^2 n r^2 + 2F^2 n^3 r^2 - F^2 n^4 r^2 \right]$$

Cartesian Approach:

$$\text{ArcCos} \left[\frac{1}{F^2 n^2 r^2} \left(F^3 n^3 - F n^3 + n^2 n^2 - 2n^3 r^2 + F^2 n^3 - n^5 n^2 - F^2 n^3 r^2 + n^2 n^2 - F^2 n^2 r^2 + n^2 n^2 \right) \right] = \frac{F^2 n^3 r^2 - F^2 n^3 r^2 + n^2 n^2 - 2n^3 r^2 + F^2 n^3 - n^5 n^2 - F^2 n^3 r^2 + n^2 n^2 - F^2 n^2 r^2 + n^2 n^2}{F^2 n^2 r^2}$$

Refer to "theta(r)--cartesian approach" and pages 5 - 8 of Robert's lab book for more information on the Cartesian approach.

Mathematica Notebook:

(named "Far Field—Dadra.nb")

```
Clear[?, ?, P?, Pr, P?, field, i, F, n, ?, dia, thick, ?min?  
????????????????????USER MODIFIED PARAMETERS????????????????????  
F?.;  
?min ? 3.2;  
n? 1.54;  
?max? 49Degree;  
?? 1? 3;  
F??r_ :? ?0.11840789` ? 46.424869` ??r2? 285.1578` ??r4? 998.6571` ??r6? 1104.0749` ??r8?  
395.89564` ??r10;  
stepsize ? ?? 1200;  
????????????????????  
???_? :? F? ?n? 1? ? ?n? Cos?????  
  
F? ?min? ?n? Cos??max?? ??n? 1?;  
dia? 2? ?min? Sin??max?;  
  
??  
?min???max?;  
dia?2??min?Sin??max?;  
??  
  
thick ? F? ?min;  
plasticthick ? F? ?min? Cos??max?;  
  
xcenter ? F??n? 1?;  
lensmin ? ?min? Cos??max?;  
If?lensmin? xcenter, Print?"STOP: A piecewise integration is needed. ?max is too great."?,?
```

```

??r_? :? ArcCos??nr2? ' ?F4? 4 F4n? 6 F4n2? 4 F4n3? F4n4? F2r2? 2 F2n r2? 2 F2n3r2? F2n4r2??
    ?F2? 2 F2n? F2n2?r2??;

```

```

Pr?r_? :? ?10^?F??r?? ?10????n? Cos???r??? ^3??F^2??n? 1?^2??n?Cos???r?? ?1??;

```

```

u??_? :? ?? dia?Sin?????

```

```

rprime?r_? :? r??dia?2?;

```

```

field??_? :? ??dia^2?2? NIntegrate?  $\frac{1}{Pr?r? ? BesselJ?0, u??? ? rprime?r?? ? r, ?r, 0, dia?2??}$ 
pwrB??_? :? 10?Log?10, field???^2?

```

```

Print?"Focal length:"?

```

```

Print?N?F, 3??

```

```

Print?"Lens diameter:"?

```

```

Print?N?dia, 3??

```

```

Print?"Lens thickness:"?

```

```

Print?N?thick, 3??

```

```

Print?"Plastic block thickness:"?

```

```

Print?N?plasticthick, 3??

```

```

stepsize ? .0002;

```

```

fieldatzero ? pwrB?0?;

```

```

For?i ? 0, i ??? 6, i ?? stepsize,

```

```

    If??fieldatzero? pwrB?i?? ? 3.0, Print?"Full Width Half Max:"?; Print?N?2?i??180???, 3??; Break??,??

```

```

For?i ? 0, i ??? 6, i ?? stepsize, If??pwrB?i?? pwrB?i? stepsize??, Print?"First Null at:"?;

```

```

    Print?N?i??180???, 3??; Break????

```

```

rayleigh ? ArcSin??1.22 ???dia?? ? Degree;

```

```

Print?"Rayleigh Criterion ?Best Distance to First Null?: "?

```

```

Print?N?rayleigh, 3??

```

```

stepsize ???1200;

```

```

output ? Table??i, pwrB?i??, ?i, 0, ??? 6, stepsize??;

```

```

ListPlot?output?

```

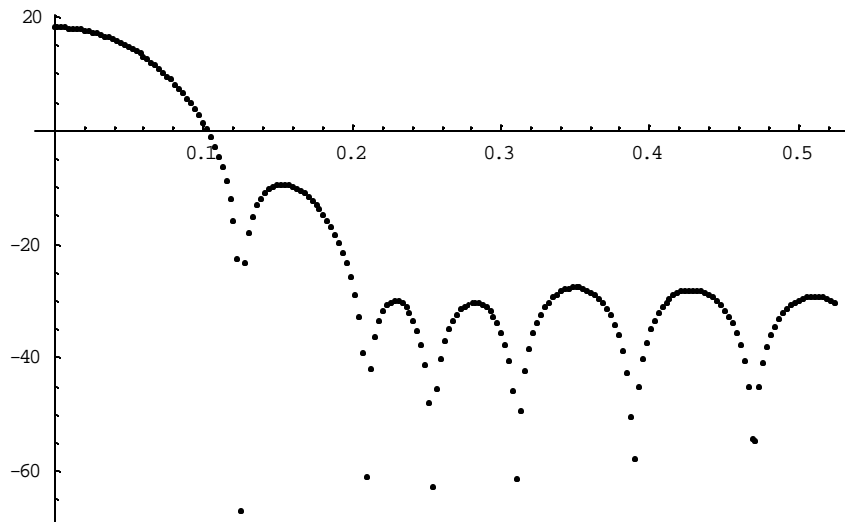
Results:

θ_{\min} (cm)	F (cm)	Diameter (cm)	Lens Thickness (cm)	Plastic Thickness (cm)	FWHM (degrees)	First Null (degrees)	Rayleigh Criterion (degrees)
3	4.92	4.53	1.91	2.94	5.39	7.22	5.15
5	8.18	7.55	3.18	4.90	3.33	4.41	3.09
7	11.46	10.57	4.46	6.87	2.41	3.21	2.21
9	14.73	13.58	5.73	8.83	1.95	2.46	1.71

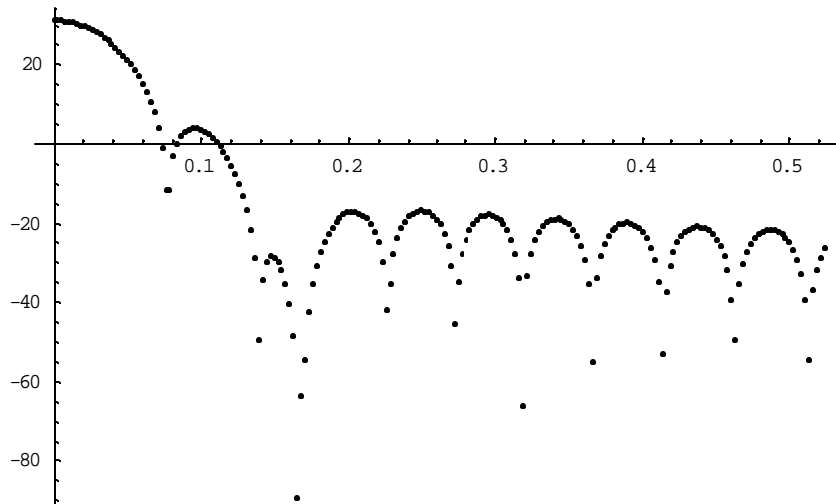
Index of refraction 1.54, Attenuation of -30dB on edges, Wavelength $1/3\text{ cm}$, Stepsize = 0.001

Beam maps of all the distances follow.

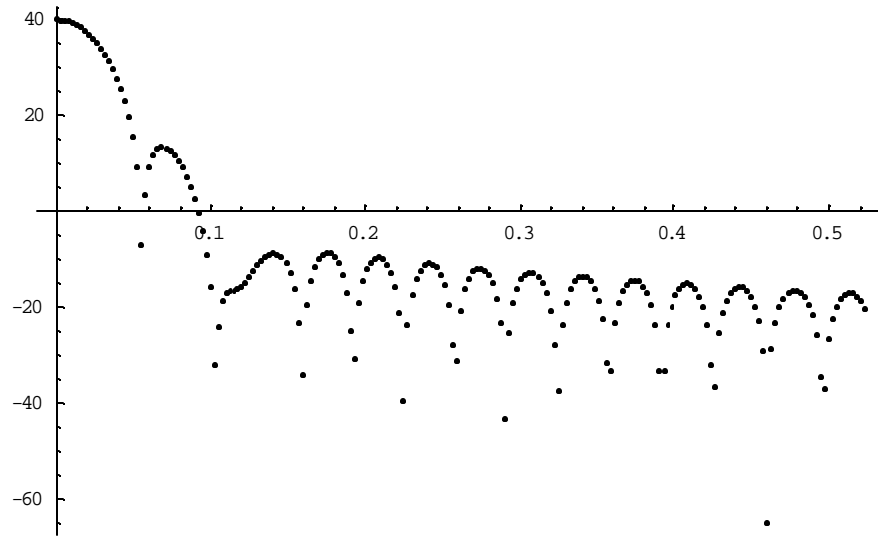
$\theta_{\min} = 3$



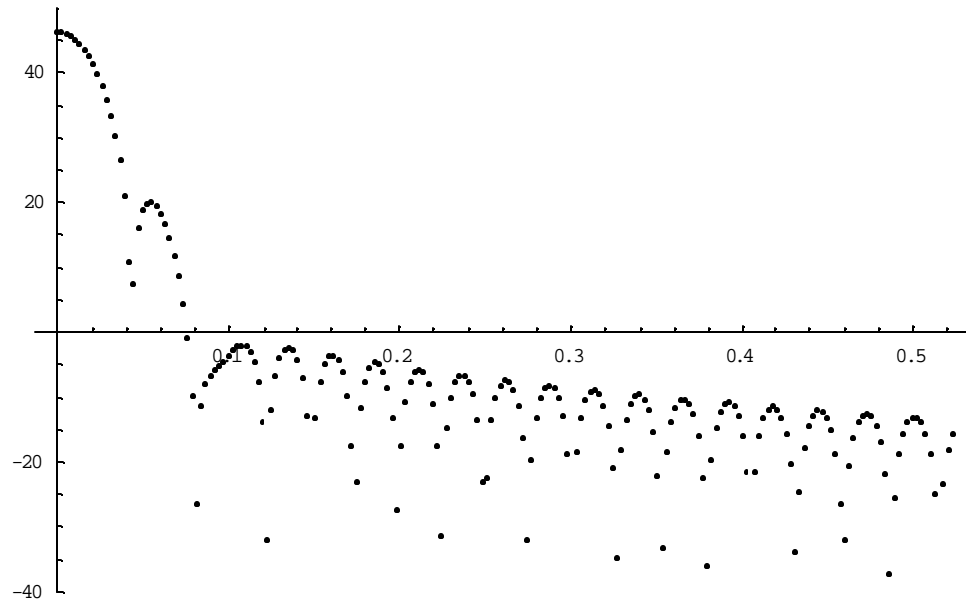
$\theta_{\min} = 5$



?min = 7



?min = 9



Choice of Plastic:

In principle, almost any material with the desired index of refraction can be used for the lens. However, in practice metal-plate lenses cannot be used because their edges cause diffraction and they introduce a polarization (not very appealing!). So, most lenses are made of dielectric materials.

There are several issues involved in choosing the material:

- ?? Constancy of the index of refraction: the index of refraction of many materials changes with frequency of radiation. We must be careful and select a material that has the required index of refraction at the frequency used.
- ?? Loss Tangent: the material must have a low loss tangent so that little power is lost due to dielectric losses.
- ?? Ease of machining: the material must be hard enough to be machined finely as the anti-reflection grooving is only a quarter wavelength thick.
- ?? Temperature Constancy: the material must be able to withstand the operating temperature without changing shape.

There are several materials that are used professionally for microwave lenses (See Appendix III). However, high-density polyethylene (HDPE) was chosen for the prototype for cost purposes (purchased from McMaster-Carr).

Anti-Reflection Grooving:

Anti-reflection grooving is used to eliminate reflection. The depth of the anti-reflection grooving is exactly 1/4 of the wavelength. Thus, the waves reflected from the top surface and the bottom surface are exactly 1/2 wavelength out of phase and exactly cancel each other out. We could not determine the spacing of this grooving.

Designing the Lens:

The polar equation for the lens has been given before in the section on the shape of the lens. This polar formula can be converted to a rectangular formula in terms of x (distance along axis) and y (distance from axis) by substituting $\sqrt{x^2 + y^2}$ for r , and $x / \sqrt{x^2 + y^2}$ for $\cos[\theta]$. Thus, the rectangular equation for the shape of the lens is:

$$y = \frac{\sqrt{F^2 - 2F^2 n^2 + F^2 n^2 - 2Fx + 2Fn^2 x^2 - n^2 x^2}}{n}$$

Abhinav's C program, which produces a table of y versus x in order to machine the lens, is included in Appendix IV.

Appendix I

```
coflr ? ??0.1612481**^01 , ?0.1863041**^01 , ?0.2184369**^01 , ?0.2496738**^01 ,
    ?0.2109388**^01 , ?0.1443323**^01 , ?0.1066749**^01 , ?0.4444911**^00 ,
    ?0.3088715**^00 , ?0.8542718**^-01 , ?0.5742379**^-01 , ?0.1129113**^-01 ,
    ?0.7453253**^-02 ?;
cofli ? ??0.2323390**^01 , ?0.2895757**^01 , ?0.3256977**^01 , ?0.3334378**^01 ,
    ?0.2689537**^01 , ?0.1800650**^01 , ?0.1228577**^01 , ?0.5368584**^00 ,
    ?0.3362545**^00 , ?0.1013582**^00 , ?0.6049709**^-01 , ?0.1326495**^-01 ,
    ?0.7698154**^-02 ?;
cof2r ? ?0.1617114**^01 , 0.1971161**^01 , 0.2276972**^01 , 0.2345550**^01 ,
    0.2041853**^01 , 0.1574014**^01 , 0.1025994**^01 , 0.5792622**^00 , 0.3123293**^00 ,
    0.1309730**^00 , 0.6259933**^-01 , 0.1998716**^-01 , 0.8872578**^-02 ?;
cof2i ? ?0.2462886**^01 , 0.3231531**^01 , 0.3487001**^01 , 0.3242393**^01 ,
    0.2601094**^01 , 0.1771790**^01 , 0.1065642**^01 , 0.5721155**^00 , 0.2715521**^00 ,
    0.1193739**^00 , 0.4712724**^-01 , 0.1731038**^-01 , 0.5947467**^-02 ?;

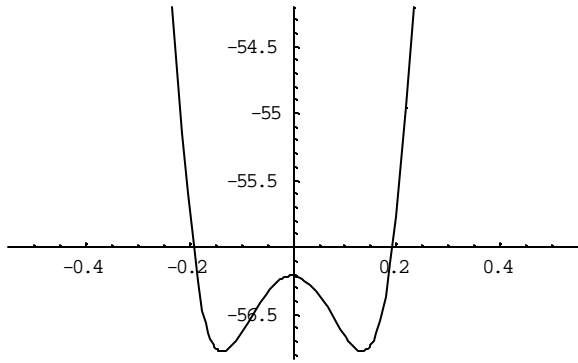
Clear ?n, ?, ?, R, h, H, Field, field, mo, ne?;
k ? 6??;
??H ? BesselJ?n, k?R? ? I?Bessely?n, k?R?;??

H? k?R?h;
h ? Sqrt?????k?R???BesselJ?n?1?2, k?R? ? I?Bessely?n?1?2, k?R??;
mo ?
    ?LegendreP ?n, 1, Cos?????1? Sin?????Cos?????hat ?
    D?LegendreP ?n, 1, Cos????, ??? Sin?????hat??h;
ne ? n??n?1???1??k?R???LegendreP ?n, 1, Cos?????Cos?????h?rhat ?
    ?D?LegendreP ?n, 1, Cos????, ??? Cos?????hat ?
    LegendreP ?n, 1, Cos?????1? Sin?????Sin?????hat??D?H, R???1??k?R??;

R? 1000;
For?field? 0; n? 1, n? 14, n??,
    field ?? ?coflr??n??? cofli??n??? I? ?mo ? ?cof2r??n??? cof2i??n??? I??ne?

field? ?field;

??? ?12;
pwr ? Abs?Coefficient?field, ?hat??^2? Abs?Coefficient?field, ?hat??^2?
    Abs?Coefficient?field, rhat??^2;
Plot?10? Log?10, pwr?, ??, ??? ?6, ? ?6??
```

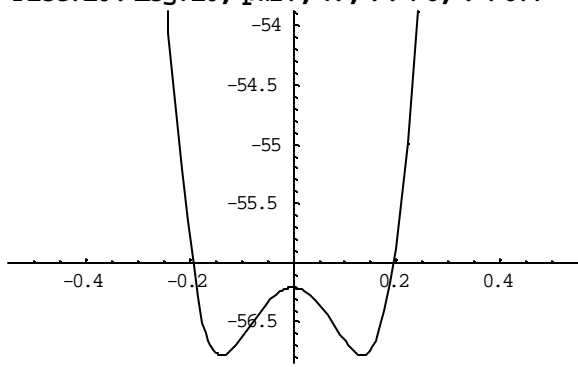


?Graphics?

??Copolar field??

pwr ? Abs?Cos??? ? Coefficient?field, ?hat?? Sin??? ? Coefficient?field, ?hat??^2;

Plot?10? Log?10, pwr?, ??, ???6, ??6??

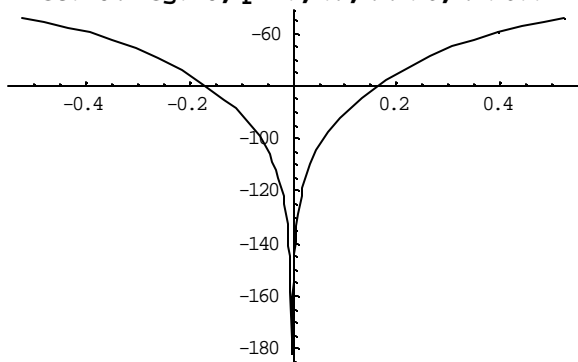


?Graphics?

??Crosspolar field??

pwr ? Abs?Sin??? ? Coefficient?field, ?hat?? Cos??? ? Coefficient?field, ?hat??^2;

Plot?10? Log?10, pwr?, ??, ???6, ??6??



Appendix II

CCRHRN produced the spherical wave expansion coefficients for Matt's horn. This data is in the files "file27_x.spw" and "file27_y.spw" (x and y polarization, respectively). The y-polarization coefficient data was given to Dadra. If the distance from the horn's phase center to the back of the lens (r_{\min}) is known, then Dadra should be used to produce a P_{θ} which is r_{\min} from the horn.

I used "code2.exe" to process Dadra's output (convert degrees to radians and confine the near-field output from -60 to 60 degrees for ease of computation) for use by Curve Expert 1.3, a curve-fitting program. Curve Expert found P_{θ} .
(see \Dadra\data for these files)

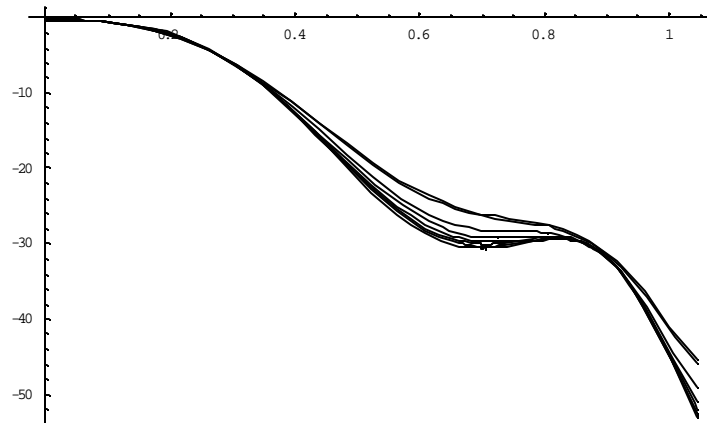
The Dadra input data file (not the .spw files) is included in the memo. Dadra's output files are named like "plot1_11.dat". This file produces the near field at an 11 cm distance. The 1 indicates the angle at which the slice was taken (perpendicular to the polarization for 1). The files produced by "code2.exe" are named like "res1_11.dat".
(see \Dadra\data for these files)

Below are the various P_{θ} we used. $P_{3\theta}$, for example, refers to P_{θ} with $r_{\min}=3$ cm. Note that before these P_{θ} can be used in the notebook "Far Field--Dadra.nb", the "?" must be replaced with "[r]" and " P_{θ} " with " $P_{\theta}[r]$ ".

Clear???

```
P3??_? :? ?0.1024103 ? 47.640501 ?2? 266.91689 ?4? 948.75735 ?6? 1054.4315 ?8? 379.47289 ?10;  
P3p2??_? :? ?0.11840789 ? 46.424869 ?2? 285.1578 ?4? 998.6571 ?6? 1104.0749 ?8? 395.89564 ?10;  
P5??_? :? ?0.2229843 ? 37.998093 ?2? 398.06116 ?4? 1300.9367 ?6? 1399.2497 ?8? 491.48743 ?10;  
P7??_? :? ?0.2901351 ? 32.347932 ?2? 465.93718 ?4? 1478.4408 ?6? 1569.1169 ?8? 545.2715 ?10;  
P9??_? :? ?0.33485915 ? 28.607781 ?2? 508.43927 ?4? 1589.0204 ?6? 1675.1242 ?8? 579.06601 ?10;  
P11??_? :? ?0.36740756 ? 25.906638 ?2? 538.16107 ?4? 1666.57 ?6? 1750.2634 ?8? 603.45127 ?10;  
P13??_? :? ?0.39304456 ? 23.829826 ?2? 560.46694 ?4? 1725.0567 ?6? 1807.6242 ?8? 622.41962 ?10;  
P17??_? :? ?0.43044091 ? 20.843589 ?2? 591.94638 ?4? 1808.255 ?6? 1890.4928 ?8? 650.43087 ?10;
```

Plot??P3???, P3p2???, P5???, P7???, P9???, P11???, P13???, P17???, ??, 0, 60 Degree??



Appendix III

1) C-STOCK .0005 (<http://www.cumingcorp.com/microdm.html>)

C-STOCK .0005 is a specialty plastic product with very low loss tangent (dissipation factor), great homogeneity of dielectric constant and loss tangent and excellent temperature stability. The material is a crosslinked polystyrene, which is a thermosetting plastic; i.e., it does not melt at elevated temperatures. C-STOCK .0005 is useful in many RF and microwave applications. It has been used extensively for microwave lenses, antenna insulators, and for a multitude of uses as supports and machined parts in waveguide and coaxial transmission lines.

C-STOCK .0005 has good optical clarity and it is readily machined using standard practices for rigid plastics machining. It may be bonded in place using most epoxy adhesives, acrylates or flexible urethane adhesives.

C-STOCK .0005 is available in 12" x 12" sheets ranging from 0.125" to 6" thickness and rods 12" long from 0.125" to 6" in diameter. Larger sheets and rods can be supplied on special order.

Typical Properties:

Loss tangent	0.0005
Dielectric constant	2.54
Homogeneity of dielectric constant	± 0.01
Volume resistivity ohm-cm	>10 ¹⁶
Dielectric strength - volts/mil	500
Thermal conductivity - BTU x in/hr x ft ² °F	0.87
Coefficient of linear expansion	35 x 10 ⁻⁶ /°F
Water absorption - weight percentage	0.003
Operating temperature range - °F	-94 through 257 continuous short time exposure up to 390
Specific gravity	1.06
Tensile strength – psi	11,000
Flexural strength – psi	17,000
Modulus of elasticity – psi	300,000

2) Rexolite (http://www.complast.com/rexolite/rexolite_shortdata.htm)

Dielectric Properties Outstanding! Maintains a Dielectric Constant of 2.53 through 500Ghz with extremely low dissipation factors. Ideal for microwave lenses, microwave circuitry, antennae, coaxial cable connectors, sound transducers, television satellite dishes and sonar lenses. Other applications include nondestructive material testing devices, surveillance equipment, radar windows, radomes and missile guidance system housings. One interesting application for radar is mapping the Earth's surface from fast aircraft at high altitudes.

Machinability Handles well in all machining operations. Tool configuration is similar to those used on Acrylic. Due to high resistance to cold flow and freedom from stress, it is easily machined or laser beam cut to very close tolerances. Accuracies of .0001 can be obtained by grinding. Crazeing can be avoided by using sharp tools and avoiding excessive heat during polishing. Stress relieving is not required before, during or after machining.

Chemical Resistance Alkalies, Alcohols, Aliphatic Hydrocarbons and mineral acids have no effect. Aromatic & chlorinated hydrocarbons cause swelling and should be avoided. Light Weight 15% lighter than Acrylic and less than half of TFE (Teflon). Specific Gravity is 1.05.

Appendix IV

machine.c

//Calculates y as a function of x for the elliptical lens shape

```
#include <stdio.h>
```

```
#include <math.h>
```

```
int main(void)
```

```
{
```

```
    double pmin, tmax, n, F, start;
```

```
    double x, y, stepsize;
```

```
    pmin = 3.2;
```

```
    tmax = 0.872665;
```

```
    n = 1.54;
```

```
    F = pmin*(n-cos(tmax))/(n-1);
```

```
    start = pmin*cos(tmax);
```

```
    stepsize = 0.01;
```

```
    for (x=start; x<=F; x+=stepsize)
```

```
    {
```

```
        y = sqrt(F*F - 2*F*F*n + F*F*n*n - 2*F*x + 2*F*n*x + x*x - n*n*x*x)/n;
```

```
        printf("%f %f\n", x, y);
```

```
    }
```

```
    return 0;
```

```
}
```

machine2.c

//Calculates y as a function of x for the spherical lens shape

```
#include <stdio.h>
```

```
#include <math.h>
```

```
int main(void)
```

```
{
```

```
    double pmin, tmax, n, F, start;
```

```
    double x, y, stepsize;
```

```
    pmin = 3.2;
```

```
    tmax = 0.872665;
```

```
    n = 1.54;
```

```
    F = pmin*(n-cos(tmax))/(n-1);
```

```
    start = pmin*cos(tmax);
```

```
    stepsize = 0.01;
```

```
    for (x=start; x<=pmin; x+=stepsize)
```

```
    {
```

```
        y = sqrt(pmin*pmin - x*x);
```

```
        printf("%f %f\n", x, y);
```

```
    }
```

```
    return 0;
```

```
}
```

¹ Olver, Clarricoats, Kishk, and Shafai. *Microwave Horns and Feeds*. IEE and IEEE Press, 1994.

² Clarricoats and Olver. *Corrugated Horns for Microwave Antennas*. Peter Peregrinus Ltd., 1984.

³ Curve Expert 1.3. <http://www.ebicom.net/~dhyams/cvxpt.html>